First Order Theories - Rationals and Reals

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Outline

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Introducton

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Integers, Rationals and Reals

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Why cannot n be an arbitrary rational in Axioms ($\mathcal{A}$9.) and ($\mathcal{A}$.10)?

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($\mathcal{A}11.$) $(\forall x)\ (x \neq 0) \rightarrow (\exists y)\ x \cdot y = 1$.

Axiom set of the Theory of Reals

Axiom set

The axiom set of $T_{\mathbb{R}}$ is the following:

($\mathcal{A}1.$) $(\forall x)(\forall y)\ [(x \geq y) \land (y \geq x)] \rightarrow (y = x)$.

($\mathcal{A}2.$) $(\forall x)(\forall y)(\forall z)\ [(x \geq y) \land (y \geq z)] \rightarrow (x \geq z)$.

($\mathcal{A}3.$) $(\forall x)(\forall y)\ (x \geq y) \lor (y \geq x)$.

($\mathcal{A}4.$) $(\forall x)(\forall y)(\forall z)\ (x + y) + z = x + (y + z)$.

($\mathcal{A}5.$) $(\forall x)\ (x + 0 = x)$.

($\mathcal{A}6.$) $(\forall x)\ (x + (-x)) = 0$.

($\mathcal{A}7.$) $(\forall x)(\forall y)\ (x + y) = (y + x)$.

($\mathcal{A}8.$) $(\forall x)(\forall y)(\forall z)\ (x \geq y) \rightarrow (x + z) \geq (y + z)$.

($\mathcal{A}9.$) $(\forall x)(\forall y)(\forall z)\ (x \cdot y) \cdot z = x \cdot (y \cdot z)$.

($\mathcal{A}10.$) $(\forall x)\ 1 \cdot x = 1$.

($\mathcal{A}11.$) $(\forall x)\ (x \neq 0) \rightarrow (\exists y)\ x \cdot y = 1$.

($\mathcal{A}12.$) $(\forall x)(\forall y)\ x \cdot y = y \cdot x$.

Axiom set of the Theory of Reals

Axiom set

The axiom set of $T_{\mathbb{R}}$ is the following:

($\mathcal{A}1.$) $(\forall x)(\forall y)\ [(x \geq y) \land (y \geq x)] \rightarrow (y = x)$.

($\mathcal{A}2.$) $(\forall x)(\forall y)(\forall z)\ [(x \geq y) \land (y \geq z)] \rightarrow (x \geq z)$.

($\mathcal{A}3.$) $(\forall x)(\forall y)\ (x \geq y) \lor (y \geq x)$.

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($\mathcal{A}9.$) $(\forall x)(\forall y)(\forall z)\ (x \cdot y) \cdot z = x \cdot (y \cdot z)$.

($\mathcal{A}10.$) $(\forall x)\ 1 \cdot x = 1$.

($\mathcal{A}11.$) $(\forall x)\ (x \neq 0) \rightarrow (\exists y)\ x \cdot y = 1$.

($\mathcal{A}12.$) $(\forall x)(\forall y)\ x \cdot y = y \cdot x$.

($\mathcal{A}13.$) $(\forall x)(\forall y)\ x \geq 0 \land y \geq 0 \rightarrow x \cdot y \geq 0$.

Axiom set (contd.)

Axiom set (contd.)

Axiom set

Axiom set (contd.)

Axiom set

($\mathcal{A}$14.) $(\forall x)(\forall y)(\forall z)\ x \cdot (y + z) = x \cdot y + x \cdot z).$

Axiom set (contd.)

Axiom set

($\mathcal{A}$14.) $(\forall x)(\forall y)(\forall z)\ x \cdot (y + z) = x \cdot y + x \cdot z)$.

($\mathcal{A}$15.) $0 \neq 1$.

Axiom set (contd.)

Axiom set

($\mathcal{A}$14.) $(\forall x)(\forall y)(\forall z)\ x \cdot (y + z) = x \cdot y + x \cdot z)$.

($\mathcal{A}$15.) $0 \neq 1$.

($\mathcal{A}$16.) $(\forall x)(\exists y)\ x = y^2 \vee -x = y^2$.

Axiom set (contd.)

Axiom set

($\mathcal{A}$14.) $(\forall x)(\forall y)(\forall z)\ x \cdot (y + z) = x \cdot y + x \cdot z)$.

($\mathcal{A}$15.) $0 \neq 1$.

($\mathcal{A}$16.) $(\forall x)(\exists y)\ x = y^2 \vee -x = y^2$.

($\mathcal{A}$17.) For each odd integer n,
$(\forall \overline{x})(\exists y)\ [y^n + x_1 \cdot y^{n-1} + \ldots x_{n-1} \cdot y + x_n = 0]$,

Axiom set (contd.)

Axiom set

($\mathcal{A}$14.) $(\forall x)(\forall y)(\forall z)\ x \cdot (y+z) = x \cdot y + x \cdot z)$.

($\mathcal{A}$15.) $0 \neq 1$.

($\mathcal{A}$16.) $(\forall x)(\exists y)\ x = y^2 \vee -x = y^2$.

($\mathcal{A}$17.) For each odd integer n,
$(\forall \overline{x})(\exists y)\ [y^n + x_1 \cdot y^{n-1} + \ldots x_{n-1} \cdot y + x_n = 0]$, where $\overline{x} = [x_1, x_2, \ldots x_n]$.