Induction - Complete Induction

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Outline

Complete Induction or Strong Induction

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Axiom Schema

$$[(\forall n)\ (\forall n')\ ((n' < n) \rightarrow P(n')) \rightarrow P(n)] \rightarrow (\forall x)\ P(x).$$

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Example

Argue that

$$(\forall x)(\forall y)\ (y > 0) \rightarrow [\text{rem}(x, y) < y].$$