Induction - Structural Induction

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Outline

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Main Idea

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(ii) Every structure is decomposable into substructures having the desired property.

(iii) The difficulty lies in proving that two substructures can be combined with the property continuing to hold. Typically, creativity is required in choosing the appropriate structures into which the given structure is to be decomposed.

Illustrative Examples

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Argue that a minimally connected graph on n vertices has exactly $(n-1)$ edges.

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Argue that every formula in propositional logic over a set of variables can be written using only the symbols $\wedge$, $\neg$ and **true**, over the same set of variables.