Propositional Logic - Basics

K. Subramani[1]

[1] Lane Department of Computer Science and Electrical Engineering
West Virginia University

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Outline

Motivation

Why Logic?

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Statement (or Atomic Proposition) -

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(i) The board is black.

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(i) The board is black.

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(iii) The moon is made of green cheese.

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Statement (or Atomic Proposition) - A sentence that is either **true** or **false**.

Example

(i) The board is black.

(ii) Are you John?

(iii) The moon is made of green cheese.

(iv) This statement is false.

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(ii) Are you John?

(iii) The moon is made of green cheese.

(iv) This statement is false. (Paradox).

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Example

$I : \{P \rightarrow \textbf{true}, Q \rightarrow \textbf{false}, \ldots\}$.

Conjunction

Semantics of Conjunction

A	B	$A \wedge B$
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**F**

Disjunction

Semantics of Disjunction

A	B	$A \vee B$
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

Negation

Semantics of Negation

A	A'
T	**F**
F	**T**

Implication

Semantics of Implication

A	B	$A \rightarrow B$
T	**T**	**T**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

Implication

Semantics of Implication

A	B	$A \rightarrow B$
T	**T**	**T**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

Note

Note that $A \rightarrow B$ is the same as $A' \vee B$. A is called the antecedent and B is the consequent of the implication.

Equivalence

Semantics of Equivalence

A	B	$A \leftrightarrow B$
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**T**

Equivalence

Semantics of Equivalence

A	B	$A \leftrightarrow B$
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**T**

Note

Note that $A \leftrightarrow B$ is the same as $(A \rightarrow B) \wedge (B \rightarrow A)$.