Discrete Mathematics 2 - Homework I

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1 Instructions

1. The homework is due on February 7, in class.
2. Each question is worth 4 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.
4. The work must be entirely your own. You are expressly **prohibited** from consulting with colleagues or the internet (with the exception of the material on the course website).

2 Problems

1. Explain the differences between the constructs $\rightarrow$ and $\Rightarrow$.
2. Prove or disprove the validity of the argument below using inference rules:

$$[P \wedge (Q \vee R)] \rightarrow [(P \wedge Q) \vee (P \wedge R)]$$

3. Prove argument validity using semantic arguments.

$$[(P \wedge Q) \rightarrow R] \rightarrow [P \rightarrow (Q \rightarrow R)]$$

4. In class we learnt that any formula in propositional logic can be represented in Conjunctive Normal Form (CNF). Furthermore, a conversion to CNF can be effected efficiently. A CNF formula is said to be k-CNF, if each clause contains at most k literals. Argue that any CNF formula can be expressed in 3-CNF, through an efficient conversion.
5. The GRAPH-COLORING problem is a well-known problem in computer science. In this problem, you are given a graph $G = \langle V, E \rangle$, with vertex set V and edge set E, with $|V| = n$ and $|E| = m$. You are given a collection of K colors $\{c_1, c_2, \ldots c_K\}$. The goal is to assign colors to the vertices satisfying the following constraints:
   (a) Each vertex is assigned exactly one color.
   (b) No two vertices connected by an edge, are assigned the same color.

   Express the above problem as an instance of CNF satisfiability.