Computational Complexity

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Outline

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1. The class **NP**
2. The class **coNP**

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R is **polynomial balanced** if $(x, y) \in R$ implies $|y| \leq |x|^k$ for some $k \geq 1$.
That is, the length of the second component is always bounded by a polynomial in the length of the first.

Class NP (contd.)

Proposition

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Now we show $L = \{x : \exists y, (x, y) \in R\}$. Since N decides L, $\forall x \in L$, there must be a y such that $(x, y) \in R$, and hence $L \subseteq \{x : \exists y, (x, y) \in R\}$; Conversely, $\forall x \in \{x : \exists y, (x, y) \in R\}$, it must be the cast that N accepts x. It means $x \in L$, and hence $\{x : \exists y, (x, y) \in R\} \subseteq L$. Thus $L = \{x : \exists y, (x, y) \in R\}$.

$\square$

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Examples

SAT: The certificate is just an assignment that satisfies the Boolean expression.
HAMILTON PATH: The certificate is precisely a Hamilton path in the graph.

SAT

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Definition

kSAT, where $k \geq 1$ is an integer, is the special case of SAT in which the formula is in CNF, and all clauses have k literals.

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Another way of looking at coNP

Just as **NP** can be considered to be the set of problems with succinct ”yes” certificates, **coNP** can be considered to be the set of problems with succinct ”no” certificates. This means that a ”no” instance of a problem in **coNP** has a short proof of it being a ”no” instance.

A **coNP** problem

Examples

1. coSAT $= \{\langle b \rangle : b$ is a boolean expression with no satisfying assignments$\}$

The class **NP** $\cap$ **coNP**

Properties

Poblems in the class **NP** $\cap$ **coNP** have both succinct “yes” and succinct “no” certificates.

Inclusion Relationships

Relation to P

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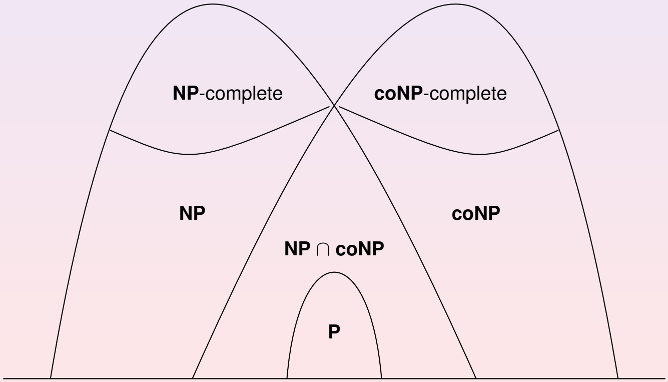
Just as $\mathbf{P} \subseteq \mathbf{NP}$, we have that $\mathbf{P} = \mathbf{coP} \subseteq \mathbf{coNP}$. Thus $\mathbf{P} \subseteq \mathbf{NP} \cap \mathbf{coNP}$.
It is unknown if $\mathbf{P} = \mathbf{NP} \cap \mathbf{coNP}$.

The Complexity Picture



A randomized algorithm for the 2SAT problem

Goal

Let $\phi = C_1 \wedge C_2 \wedge \ldots \wedge C_m$ denote a boolean formula in CNF over the boolean variables $\{x_1, x_2, \ldots, x_n\}$, such that each clause C_i has exactly two variables. Determine whether ϕ is satisfiable.

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2SAT can be solved in $O(m+n)$ time using Tarjan's connected components algorithm.

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Function SATISFIABILITY-TESTING(ϕ)

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end while
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Algorithm 5: Papadimitrious's randomized algorithm for 2CNF Satisfiability

Mathematical Preliminaries

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Theorem (Markov)

Let X be a non-negative random variable and let $c > 0$ denote a constant. Then $\mathbf{Pr}(X \geq c \cdot \mathbf{E}[X]) \leq \frac{1}{c}$.

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$$\begin{aligned} t(0) &= 0 \\ t(n) &= 1 + t(n-1) \\ t(i) &\leq \frac{1}{2}t(i-1) + \frac{1}{2}t(i+1) + 1, 0 < i < n \end{aligned}$$

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The above system can be solved to get $t(n) \leq n^2$. From Markov's inequality it follows that the probability that T is not transformed into $\hat{T}$ in at most $2 \cdot n^2$ flips is less than one-half.

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Randomized Complexity Classes

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Definition

The class **RP** consists of all languages $L \subseteq \Sigma^*$ that have a randomized algorithm $\mathscr{A}$ running in worst-case polynomial time, such that for any input $x \in \Sigma^*$,

- $x \in L \Rightarrow \mathbf{Pr}[\mathscr{A}(x)] = \text{``}yes\text{''}] \geq \frac{1}{2}$.
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(iv) *2SAT is in* **RP**.

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A language $L \subseteq \Sigma^*$ is in **ZPP** is it is in **RP** $\cap$ **coRP**.

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(iii) $\mathbf{RP} \subseteq \mathbf{BPP}$.

The Complexity Picture