Approximation Algorithms - Homework III

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1 Instructions

1. The homework is due on April 10.
2. Each question is worth 4 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.
4. The work must be entirely your own, although you are encouraged to discuss your approaches with your colleagues and the instructor. You are expressly **prohibited** from consulting the internet.

2 Problems

1. Consider the following problem, called the *Subset-sum ratio problem*:
   Given n positive integers, $a_1 < a_2 < \ldots < a_n$, find two disjoint non-empty subsets $S_1, S_2 \subseteq \{a_1, a_2, \ldots a_n\}$, with $\sum_{a_i \in S_1} a_i \geq \sum_{a_j \in S_2} a_j$, such that the ratio $\frac{\sum_{a_i \in S_1} a_i}{\sum_{a_j \in S_2} a_j}$ is minimized.

   Find an FPTAS for the above problem.

2. Obtain the optimal solutions to the following linear program and its dual:

$$\max \ 10 \cdot x_1 + 6 \cdot x_2 - 4 \cdot x_3 + x_4 + 12 \cdot x_5$$

   subject to

$$2 \cdot x_1 + x_2 + x_3 + 3 \cdot x_5 \leq 18$$
$$x_1 + x_2 - x_3 + x_4 + 2 \cdot x_5 \leq 6$$
$$x_i \geq 0, i = 1, 2, 3, 4, 5.$$

   You are only permitted to use the graphical method of solving linear programs and the theory of duality.

3. Use the Max-Flow Min-Cut theorem to derive the following theorem, known as Menger's theorem.

   Theorem 2.1 *Let $\mathbf{G} = \langle V, E \rangle$ be a directed graph with $s, t \in V$. Then the maximum number of edge-disjoint (vertex-disjoint) $s - t$ paths is equal to the minimum number of edges (vertices) whose removal disconnects s from t.*

4. A bin-packing algorithm is said to be *monotonic*, if the number of bins it uses for packing a subset of the items, is at most the number of bins it uses for packing all n items. Argue that the Next-Fit algorithm is monotonic, while the First-Fit algorithm is not.

5. (a) Consider the greedy algorithm for Set Cover discussed in class. Provide an example in which the dual solution $price(e)$ for each element e, computed as per the greedy algorithm, overpacks some sets S by a factor of $H_{|S|}$.

   (b) Give a counterexample to the following claim: A set cover instance, in which each element is in exactly f sets, has an $\frac{1}{f}$-integral solution, i.e., a solution in which each set is picked $(c \cdot \frac{1}{f})$ times, where $c \in \mathcal{Z}_+$.