Computational Complexity - Homework III

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1 Instructions

1. The homework is due on April 10.
2. Each question is worth 4 points.
3. Attempt as many problems as you can. You will be given partial credit, as per the policy discussed in class.

2 Problems

1. Assume that you have an oracle for the decision version of the Graph k-coloring problem. Show that you can *find* a k-coloring if it exists on an input graph, using at most a polynomial number of queries to this oracle.
2. In class, the notion of **NP-completeness** was discussed extensively. Is there an analogous notion of **P-completeness**? If so, what type of restriction would you place on reductions? What is the hardest problem in **P** according to your definition? What is its running time?
3. Reduce SUBSET-SUM to INTEGER PARTITIONING. Design a polynomial time algorithm for SUBSET-SUM, when the input sequence is *super-increasing*. Recall that a sequence of integers $a_1, a_2, \ldots a_n$ is said to be super-increasing, if each element is greater than the sum of all previous elements in the sequence.
4. Analogous to Conjunctive Normal Form (CNF), there exists a form called Disjunctive Normal Form (DNF). A DNF formula consists of disjuncts of conjuncts. For instance, the formula

   $$(x_1 \wedge \bar{x_2} \wedge x_3) \vee (\bar{x_1} \wedge x_4 \wedge \bar{x_5})$$

   is in DNF.

   Prove the following:

   (a) DNF Satisfiability is in **P**.

   (b) Any formula in CNF can be converted into a DNF formula on the same variables.

   Can we conclude that **P**$=$**NP**? Explain why or why not.
5. In class, we showed that the HornSAT and 2SAT problems were in **P**. Consider the following variant of SAT called MixedHornSAT: Every clause is either Horn or in 2CNF form. Argue that MixedHornSAT is **NP-complete**.