Robust Optimization: Applications

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Outline

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1. Overview
   - Theory of Robust Optimization
   - Applications of Robust Optimization
2. Applications
   - Shortest Path Problem
   - Facility Location Problem
   - Portfolio Selection Problem

Chinese proverb

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- In real-world applications of optimization, even a small uncertainty in the data can make the nominal optimal solution to the problem completely meaningless from a practical viewpoint.
- Consequently, in optmization, there exist a real need of a methodology capable of detecting cases when data uncertainty can heavily effect the quality of the nominal solution.

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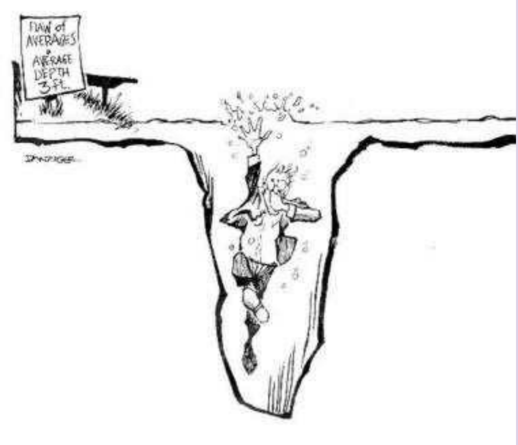

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$$3.030 \cdot 1000 - 2.999 \cdot 1000 = 31 \quad > \quad 1$$

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  - Sometimes is too pessimistic.

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- Uncertain LP form:

$$\begin{aligned} \min z &= \widetilde{\mathbf{c}} \cdot \mathbf{x} \\ \widetilde{\mathbf{A}} \cdot \mathbf{x} &\leq \widetilde{\mathbf{b}} \end{aligned}$$

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- Chance Constrain:

$$\mathbf{P}(\widetilde{\mathbf{A}} \cdot \mathbf{x} \quad > \quad \mathbf{b}) \leq \mathcal{E}$$

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$$(\mathbf{A}^{\mathbf{0}} + \sum_{j=1}^{N} \mathbf{A_j} \cdot \widetilde{\mathbf{z}}_{\mathbf{j}}) \cdot x \leq \mathbf{b}$$

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LP

Ellipsodial

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CQP

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- $\Omega = 0$ means no uncertainty and $\Omega = \sqrt{N}$ is worse case budget.

$$\mathcal{E} = exp(-\frac{\Omega^2}{2})$$

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- Electrical Engineering:
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- Finance:
  - Robust portfolio optimization: random returns;
  - Robust risk management: uncertainty of Value at Risk.

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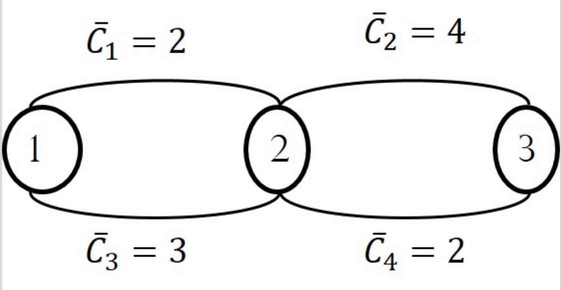

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where:

$$\widetilde{z_1}^2 + \widetilde{z_2}^2 \leq \Omega$$
$$\Omega \in (0, 1, 2)$$

Example

Ω=0	$\tilde{y}_{k1}$	$\tilde{y}_{k1}$	ROBUST SP	CLASSICAL SP
1-2	0	0	6	6
1-4	0	0	4	4
3-2	0	0	7	7
3-4	0	0	5	5

Example

Ω=1	$\tilde{y}_{k1}$	$\tilde{y}_{k1}$	ROBUST SP	CLASSICAL SP
1-2	$\sqrt{2}/_2$	$\sqrt{2}/_2$	8.83	6
1-4	$\sqrt{2}/_2$	$\sqrt{2}/_2$	8.24	4
3-2	$\sqrt{2}/_2$	$\sqrt{2}/_2$	9.82	7
3-4	$\sqrt{2}/_2$	$\sqrt{2}/_2$	7.82	5

Example

Ω=2	$\tilde{y}_{k1}$	$\tilde{y}_{k1}$	ROBUST SP	CLASSICAL SP
1-2	1	1	10.00	6
1-4	1	1	10.00	4
3-2	1	1	11.00	7
3-4	1	1	9.00	5

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Application of Facility Location Models

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- Bathroom location in a facility etc..

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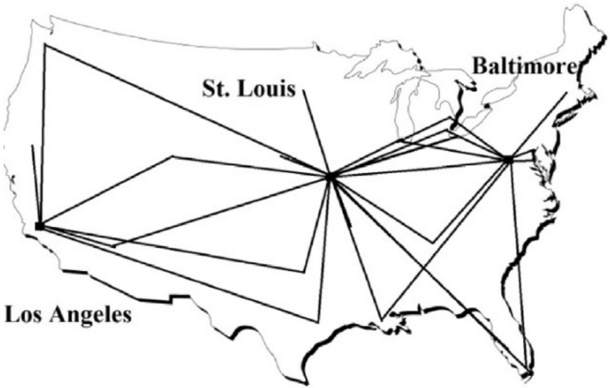
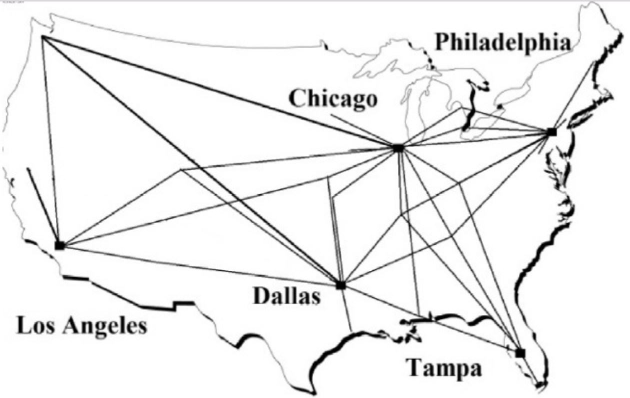
- Where demand is uncertain and its distribution is not fully specified.

Example



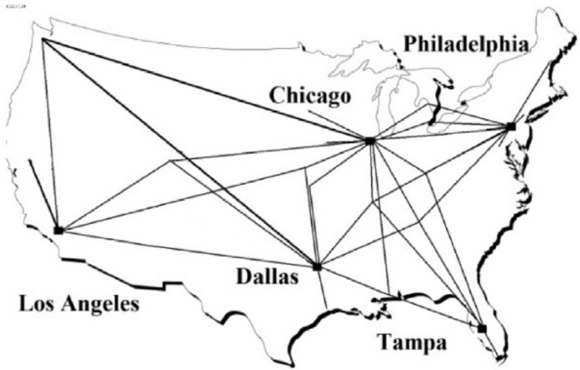
(a):Base hub location
Total Cost: 1336.19

Example



(b):Robust hub location, Ω=1.5
Total Cost: 1450.58

Example



(c):Robust hub location, **Ω=2**
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$TE(x)$ represents the tracking error of the portfolio with respect to the half-and-half benchmark.

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$$TE = \sqrt{\begin{bmatrix} x_1 - 0.5 \\ x_2 - .05 \\ x_3 \end{bmatrix}^T \begin{bmatrix} 0.1764 & 0.09702 & 0 \\ 0.09702 & 0.1089 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 - 0.5 \\ x_2 - .05 \\ x_3 \end{bmatrix}}$$

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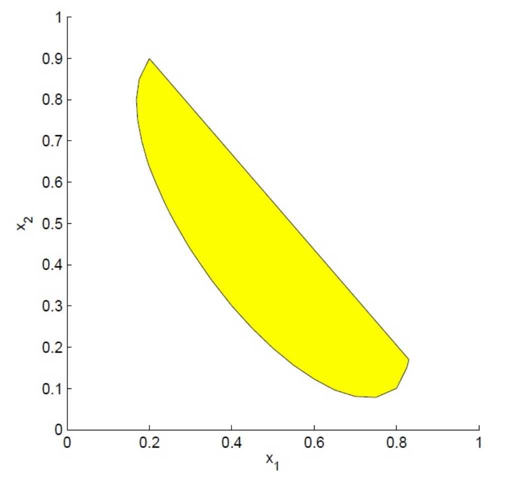

Figure: The feasible set of the portfolio selection problem

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- Scenario 1: 5.662

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- Scenario 1: 5.662
- Scenario 2: 5.662

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- Scenario 1: 5.662
- Scenario 2: 5.662
- Scenario 3: 5.000

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Relative robust formulation:

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5.662 - (4x_1 + 6x_2) &\leq t \\
5.000 - (5x_1 + 5x_2) &\leq t \\
TE(x_1, x_2, x_3) &\leq 0.10 \\
x_1 + x_2 + x_3 &= 1
\end{aligned}
$$

Example

Relative robust formulation:

$$
\begin{aligned}
\min_{x,t} \quad & t \\
5.662 - (6x_1 + 4x_2) &\leq t \\
5.662 - (4x_1 + 6x_2) &\leq t \\
5.000 - (5x_1 + 5x_2) &\leq t \\
TE(x_1, x_2, x_3) &\leq 0.10 \\
x_1 + x_2 + x_3 &= 1 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}
$$

Example

choosing a maximum tolerable regret level of 0.75 we get the following feasibility problem:

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$$\begin{aligned} \textit{Find} \quad & x \\ 5.662 - (6x_1 + 4x_2) \quad & \leq \quad 0.75 \end{aligned}$$

Example

choosing a maximum tolerable regret level of 0.75 we get the following feasibility problem:

$$\begin{aligned} \textit{Find} \quad & x \\ 5.662 - (6x_1 + 4x_2) &\leq 0.75 \\ 5.662 - (4x_1 + 6x_2) &\leq 0.75 \end{aligned}$$

Example

choosing a maximum tolerable regret level of 0.75 we get the following feasibility problem:

$$\begin{aligned} \textit{Find} \quad & x \\ 5.662-(6x_1+4x_2) &\leq 0.75 \\ 5.662-(4x_1+6x_2) &\leq 0.75 \\ 5.000-(5x_1+5x_2) &\leq 0.75 \end{aligned}$$

Example

choosing a maximum tolerable regret level of 0.75 we get the following feasibility problem:

$$\begin{aligned} \text{Find} \quad & x \\ 5.662 - (6x_1 + 4x_2) & \leq 0.75 \\ 5.662 - (4x_1 + 6x_2) & \leq 0.75 \\ 5.000 - (5x_1 + 5x_2) & \leq 0.75 \\ TE(x_1, x_2, x_3) & \leq 0.10 \end{aligned}$$

Example

choosing a maximum tolerable regret level of 0.75 we get the following feasibility problem:

$$\begin{aligned}
\textit{Find} \quad & x \\
5.662 - (6x_1 + 4x_2) &\leq 0.75 \\
5.662 - (4x_1 + 6x_2) &\leq 0.75 \\
5.000 - (5x_1 + 5x_2) &\leq 0.75 \\
TE(x_1, x_2, x_3) &\leq 0.10 \\
x_1 + x_2 + x_3 &= 1
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Example

choosing a maximum tolerable regret level of 0.75 we get the following feasibility problem:

$$\begin{aligned}
\textit{Find} \quad & x \\
5.662 - (6x_1 + 4x_2) &\leq 0.75 \\
5.662 - (4x_1 + 6x_2) &\leq 0.75 \\
5.000 - (5x_1 + 5x_2) &\leq 0.75 \\
TE(x_1, x_2, x_3) &\leq 0.10 \\
x_1 + x_2 + x_3 &= 1 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}$$

Example

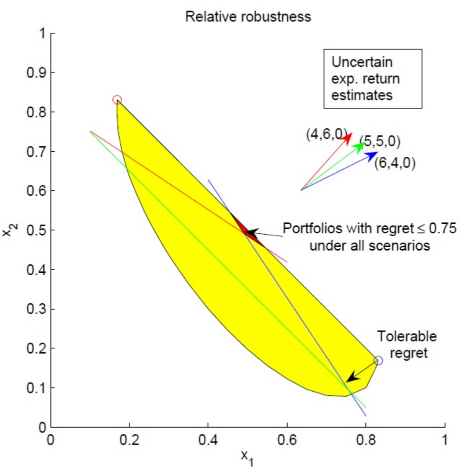

Figure: Set of solutions with regret less than 0.75