Stochastic Programming Models and application

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Outline

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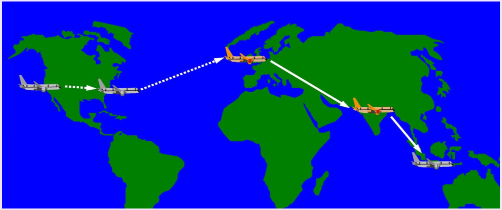
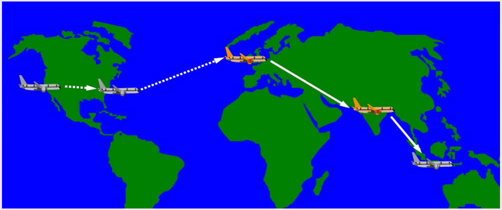
Brief Introduction



Uncertainty is Everywhere



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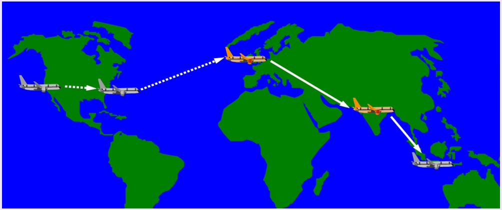

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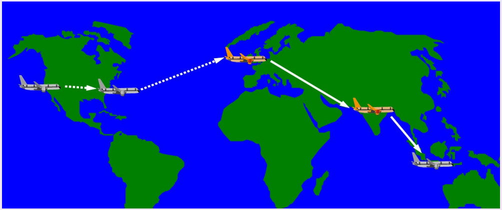

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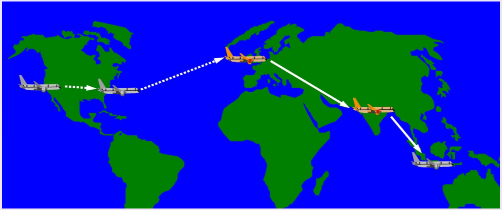

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- How much is its effect on Control variables like angle, velocity, acceleration?

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- For investors, risk is about the odds of losing money, and VaR is based on that common-sense fact. By assuming investors care about the odds of a really big loss, VaR answers the question, ”What is my worst-case scenario?” or ”How much could I lose in a really bad month?”

Methods of Calculating VaR

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  - Second method based on Variance and CoVariance method
  - Third Method based on Monte-Carlo Simulation

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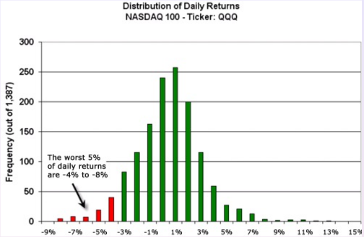
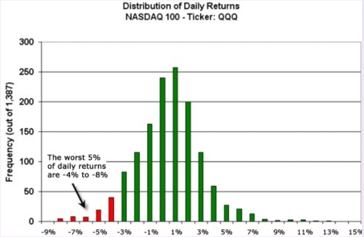


Figure: daily return for the QQQ graph

Graph Explanation

- At the highest point of the histogram (the highest bar), there were more than 250 days when the daily return was between 0 % and 1 %.



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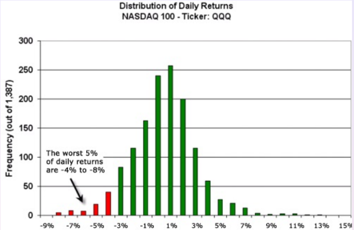
- At the far right, you can barely see a tiny bar at 13 %; it represents the one single day (in Jan 2000) within a period of five-plus years when the daily return for the QQQ was a stunning 12.4 %!



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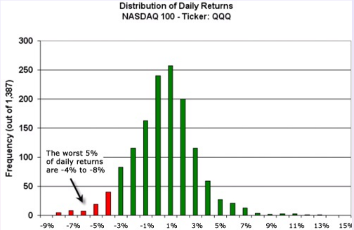

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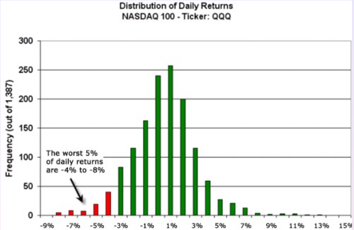

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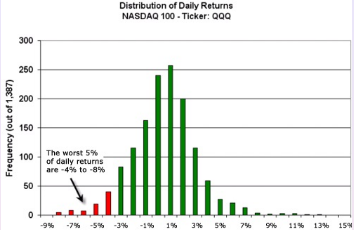

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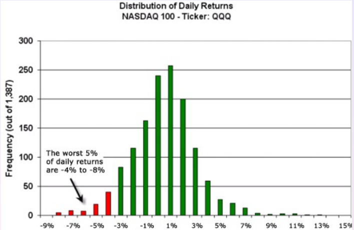

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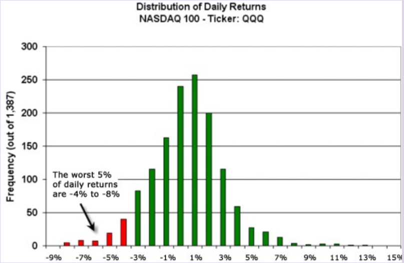

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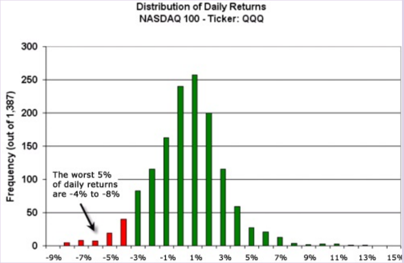

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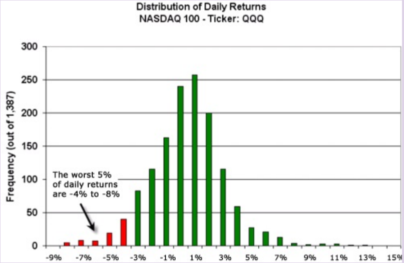

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- By 95 % confident our worst daily loss will not exceed 4$ (100 $ * -4 % = 4 $).
- We know that this answer does not express absolute certainty but instead makes a probabilistic estimate.

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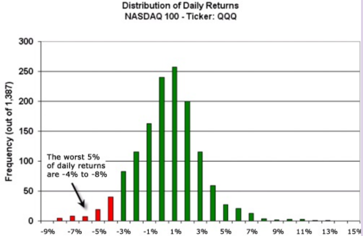

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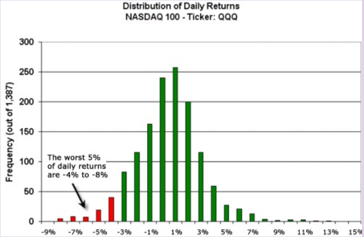

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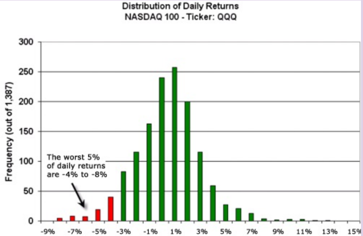

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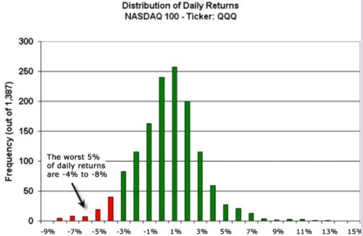

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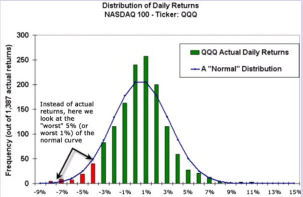

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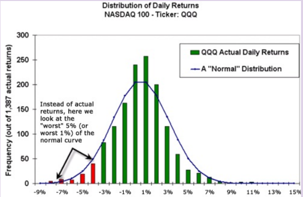

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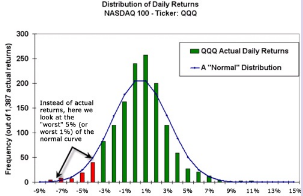

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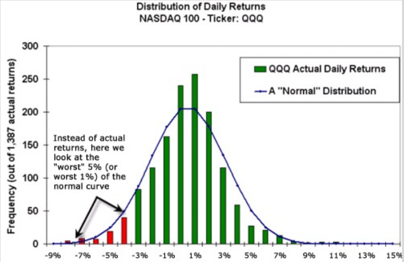
- In comparison with historical data we use the familiar curve instead of actual data.

Example


Distribution of Daily Returns
NASDAQ 100 - Ticker: QQQ
Frequency (out of 1,387 actual returns)
300
250
200
150
100
50
0
-9%
-7%
-5%
-3%
-1%
1%
3%
5%
7%
9%
11%
13%
15%
QQQ Actual Daily Returns
A "Normal" Distribution
Instead of actual returns, here we look at the "worst" 5% (or worst 1%) of the normal curve

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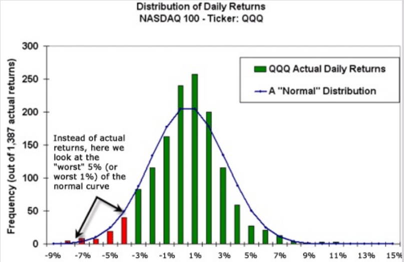

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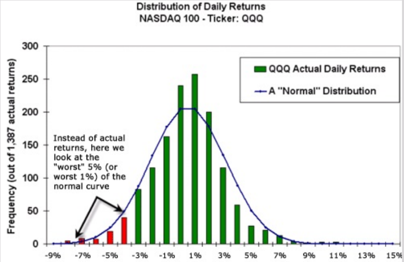

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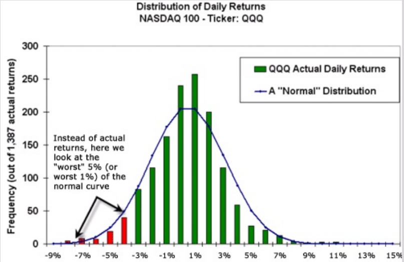

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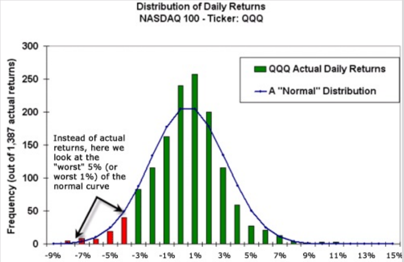

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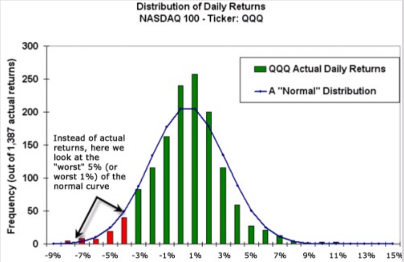

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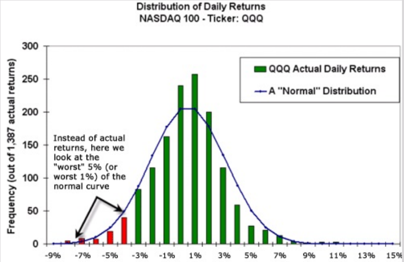

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  - For 99 % confidence the VaR is $-2.58 \cdot \sigma = -6.81\%$

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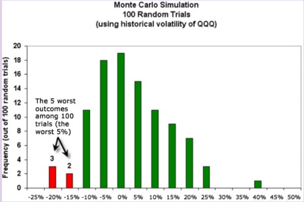
Monte-Carlo



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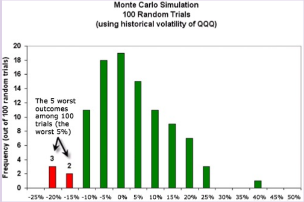

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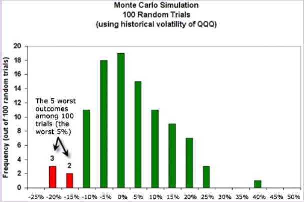

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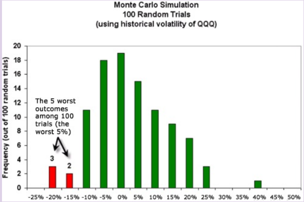

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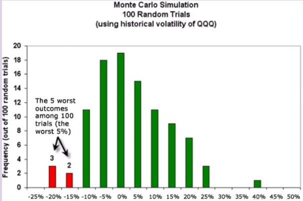

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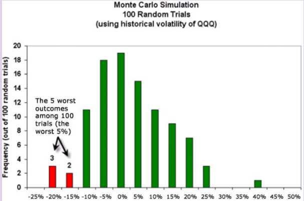

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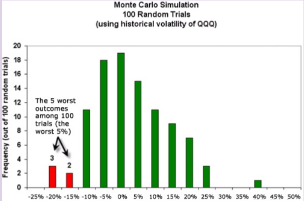

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Example

	Investment	VAR Method	Standard Deviation	Time Period	Calculated VAR
1	QQQ	Historical	N/A *	Daily	~ - 4.0%
2	QQQ	Variance-Covariance	2.64%	Daily	- 6.16%
3	QQQ	Monte Carlo simulation	N/A *	Monthly	- 15%

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Confidence Level	The Maximum Loss--below the Expected or Average Return--as a Function of Standard Deviation (σ) and Time (T)
95% confidence	$-1.65 \times \sigma \times \sqrt{T}$
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95% confidence	$-1.65 \times 2.64\% \times \sqrt{20} = -19.5\%$
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$$max_x \mu^T x$$

$$s.t. \quad VaR_\alpha \leq U_{\alpha \cdot j}, j = 1, ..., j$$

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  - The Monte Carlo simulation is complex, but has the advantage of allowing users to tailor ideas about future patterns that depart from historical patterns.

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- The objective of the model is to allocate funds among available assets to maximize expected wealth at the end of the planning horizon T less expected penalized shortfalls accumulated through the planning horizon:

  c_t = Piecewise linear convex cost function.

Solution

$$\max \quad E\Big[\textstyle\sum_i x_{iT} - \sum_{t=1}^{T} c_t(w_t)\Big]$$

subject to

asset accumulation:
$$\textstyle\sum_i x_{it} - \sum_i (1 + RP_{it} + RI_{it}) x_{i,t-1} = F_t - P_t - I_t \quad \text{for } t = 1, \dots, T,$$

interest income shortfall:
$$\textstyle\sum_i RI_{it} x_{i,t-1} + w_t - v_t = g_t L_{t-1} \quad \text{for } t = 1, \dots, T,$$

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- Liability balances and cash flows are computed so as to satisfy the liability accumulation relations: $L_t = (1 + g_t)L_{t-1} + F_t - P_t - I_t$ for $t \geq 1$.

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- Creation of scenario inputs is made in stages using a tree. The tree structure can be described by the number of branches at each stage.
- Solving this model yielded extra income estimated to be about US $80 million per year for the company.

Synthetic Options

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- Example: you can create a synthetic stock by purchasing a call option and simultaneously selling a put option on the same stock.

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- When μ = 1, the objective is to maximize the expected return.

example

Example

- Consider an investor with initial wealth $W_0 = 1$ who wants to construct a portfolio comprising one risky asset and one riskless asset using the "synthetic option" model described above.
- We write the model for a two-period planning horizon, i.e., $T = 2$. The return on the riskless asset is R per period.
- For the risky asset, the return is R_1^+ with probability 0.5 and R_1^- with the same probability at time $t = 1$.
- Similarly, the return of the risky asset is R_2^+ with probability 0.5 and R_2^- with the same probability at time $t = 2$.
- The transaction cost for purchases and sales of the risky asset is Θ.

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- The linear program for this problem is:

$$
\begin{array}{rl}
\max & 0.25z_3 + 0.25z_4 + 0.25z_5 + 0.25z_6 + \mu Z \\
\text{subject to} & \\
\text{initial portfolio:} & \alpha_0 + x_0 = 1 \\
\text{rebalancing constraints:} & x_1 = R_1^+ x_0 + A_1 - D_1 \\
 & \alpha_1 = R\alpha_0 - (1+\theta)A_1 + (1-\theta)D_1 \\
 & x_2 = R_1^- x_0 + A_2 - D_2 \\
 & \alpha_2 = R\alpha_0 - (1+\theta)A_2 + (1-\theta)D_2 \\
\text{payoff:} & z_3 + Z = R\alpha_1 + (1-\theta)R_2^+ x_1 \\
 & z_4 + Z = R\alpha_1 + (1-\theta)R_2^- x_1 \\
 & z_5 + Z = R\alpha_2 + (1-\theta)R_2^+ x_2 \\
 & z_6 + Z = R\alpha_2 + (1-\theta)R_2^- x_2 \\
\text{nonnegativity:} & \alpha_i, x_i, z_i, A_i, D_i \geq 0.
\end{array}
$$

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