

Math 93 Intermediate Algebra

Key Concepts in Fractions

Any arithmetic number can be written as a fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$

Preliminary Concepts:

- For any integer a , $\frac{0}{a} = 0$ and $\frac{a}{0}$ is undefined.
- For any integer a , $\frac{a}{1} = a$.
- For any integer a , $\frac{a}{a} = 1$
- We can get equivalent fractions from a fraction by multiplying numerator and denominator by any number.
 - Example 1: $\frac{2}{3} = \frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$
 - Example 2: $4 = \frac{4 \times 3}{1 \times 3} = \frac{12}{3}$
- To reduce or simplify a fraction, first factor the numerator and the denominator into prime factors and then cancel the factors that are common to the numerator and the denominator. The resulting fraction is called the fraction in its lowest term.
 - Example 1: Simplify $\frac{36}{24}$
$$\frac{36}{24} = \frac{\cancel{2} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{3}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{2}} = \frac{\cancel{2} \cdot \cancel{3} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{3} \cdot \cancel{2}} \cdot \frac{3}{2} = \frac{3}{2}$$
 - Example 2: Simplify $\frac{18}{72}$
$$\frac{18}{72} = \frac{\cancel{2} \cdot \cancel{9}}{\cancel{8} \cdot \cancel{9}} = \frac{2}{8} = \frac{\cancel{1} \cdot \cancel{2}}{\cancel{4} \cdot \cancel{2}} = \frac{1}{4}$$
- Now you try these:
 - $\frac{48}{62} =$
 - $\frac{24}{74} =$
- If the absolute value of the numerator is larger than the absolute value of the denominator, then the fraction is called improper. In this case, we can actually divide the numerator by the denominator and write the improper fraction as a mixed number. Similarly, we can express a mixed number as an improper fraction by multiplying the whole number part with the denominator and adding the numerator.

- Example 1: $\frac{11}{3} = 3\frac{2}{3}$
- Example 2: $\frac{21}{4} = 5\frac{1}{4}$
- Example 3: $3\frac{2}{5} = \frac{17}{5}$
- Example 4: $\frac{18}{7} = 2\frac{4}{7}$
- Now you try these:
 - $\frac{41}{3} =$
 - $4\frac{1}{3} =$
- The following worksheet is for your practice:

Multiplying and Dividing Arithmetic Fractions

- To multiply two fractions, we multiply the numerators to get the new numerator and multiply the denominators to get the new denominator. Then we reduce the resulting fraction to its simplest form.
 - $\frac{a}{b} \times \frac{c}{d} = \frac{a.c}{b.d}$
 - Example 1: $\frac{5}{6} \times \frac{9}{25} = \frac{5.9}{6.25} = \frac{\cancel{5}.3.3}{\cancel{3}.2.\cancel{5}.5} = \frac{3}{10}$
 - Example 2: $\frac{15}{16} \times \frac{8}{5} = \frac{15.8}{16.5} = \frac{\cancel{5}.3.\cancel{8}}{\cancel{2}.\cancel{8}.\cancel{5}} = \frac{3}{2}$
- Now you try these:
 - $\frac{5}{8} \times \frac{6}{25} =$
 - $3\frac{3}{4} \times 2\frac{2}{3} =$
- Two numbers whose product is 1 are called reciprocals or multiplicative inverses.
 - Reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$ because $\frac{3}{2} \times \frac{2}{3} = \frac{\cancel{3}.\cancel{2}}{\cancel{2}.\cancel{3}} = 1$
 - Reciprocal of $\frac{1}{9}$ is 9. (Why?)
 - Reciprocal of 4 is $\frac{1}{4}$ (Why?)

- To divide two fractions, we multiply the first fraction by the reciprocal of the second fraction. that is, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$
 - Example 1: $\frac{2}{3} \div \frac{7}{5} = \frac{2}{3} \times \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$
 - Example 2: $\frac{2}{3} \div \frac{4}{9} = \frac{2}{3} \times \frac{9}{4} = \frac{2 \cdot 9}{3 \cdot 4} = \frac{2 \cdot 3 \cdot 3}{3 \cdot 2 \cdot 2} = \frac{3}{2}$ or $1 \frac{1}{2}$
 - Example 3: $\frac{10}{21} \times \frac{24}{25} = \frac{10 \times 24}{21 \times 25} = \frac{2 \cdot 5 \cdot 3 \cdot 8}{3 \cdot 7 \cdot 5 \cdot 5} = \frac{16}{35}$
 - Example 3: $4 \frac{2}{5} \times 1 \frac{7}{8} = \frac{22}{5} \times \frac{15}{8} = \frac{22 \times 15}{5 \times 8} = \frac{2 \cdot 11 \cdot 3 \cdot 5}{5 \cdot 2 \cdot 4} = \frac{33}{4} = 8 \frac{1}{4}$
- Now you try these:
 - $\frac{3}{10} \div \frac{3}{5} =$
 - $9 \frac{1}{6} \div 5 =$

Adding and subtracting arithmetic fractions:

- Adding/subtracting fractions with same denominator:

To add/subtract fractions with same denominator, add/subtract the numerators and keep the same denominator as common denominator. Then, simplify the fraction to its lowest term.

 - Example 1: $\frac{5}{8} + \frac{1}{8} = \frac{5+1}{8} = \frac{6}{8} = \frac{3 \cdot 2}{4 \cdot 2} = \frac{3}{4}$
 - Example 2: $\frac{7}{9} - \frac{4}{9} = \frac{7-4}{9} = \frac{3}{9} = \frac{1}{3}$
- Now you try these:
 - $\frac{5}{9} + \frac{2}{9} =$
 - $\frac{19}{20} - \frac{7}{20} =$
- Adding/subtracting fractions with different denominators:

If the denominators are different, then we must:

 - Find the least common denominator (LCD).
 - Write each fraction as an equivalent fraction with the LCD
 - Add/subtract the numerator and keep the LCD as the common denominator.
 - Simplify and reduce to lowest terms

○ Example 1: $\frac{3}{4} + \frac{1}{6}$

LCD of 4 and 6 is 12

$$\frac{3}{4} + \frac{1}{6} = \frac{3(3)}{4(3)} + \frac{1(2)}{6(2)} = \frac{9}{12} + \frac{2}{12}$$

$$\frac{9}{12} + \frac{2}{12} = \frac{9+2}{12} = \frac{11}{12}$$

○ Example 2: $\frac{4}{5} - \frac{2}{3}$

LCD of 3 and 5 is 15

$$\frac{4}{5} - \frac{2}{3} = \frac{4(3)}{5(3)} - \frac{2(5)}{3(5)} = \frac{12}{15} - \frac{10}{15}$$

$$\frac{12}{15} - \frac{10}{15} = \frac{12-10}{15} = \frac{2}{15}$$

• Now you try these:

○ $\frac{11}{12} - \frac{5}{8} =$

○ $4\frac{2}{5} + 1\frac{7}{15} =$