Adjacent-Channel Interference in Frequency-Hopping Ad Hoc Networks

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Outline

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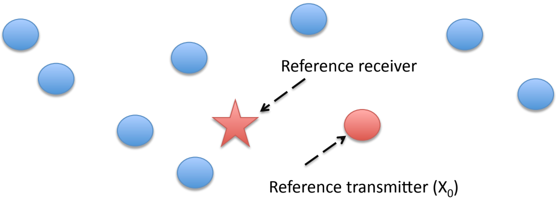
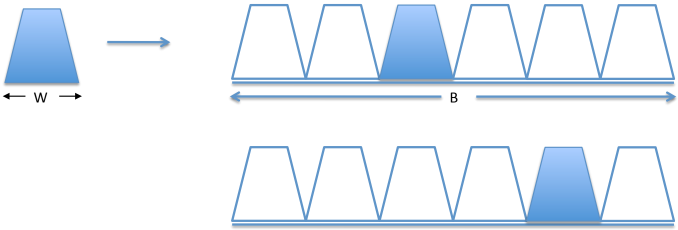
Ad Hoc Networks



- Mobile transmitters are randomly placed in a 2-D finite space.
- A reference receiver is located at the origin.
- X_i represents the i^{th} transmitter and its location.
  - X_0 is location of the reference transmitter.
  - M interfering transmitters, $\{X_1, ..., X_M\}$.
  - $|X_i|$ is distance from i^{th} transmitter to the reference receiver.
- Spatial model
  - Fixed number of interferers, placed independently and uniformly over the network area (BPP).

Frequency Hopping

- To manage interference, *frequency hopping* (FH) is used:



- Each mobile transmits with *duty factor* D.
  - Likelihood of a transmission is $D \leq 1$.
- Each transmitting mobile randomly picks from among L frequencies.
  - Probability of a *co-channel collision* is $p_c = D/L$.

Adjacent-Channel Interference

- We assumed that all interference is co-channel interference.
  - However, FSK modulation is not completely bandlimited.
  - Some of the spectrum splatters into adjacent channels.
- Typically, the frequency channels are matched to the 99-percent bandwidth of the modulation.
  - Percent of the signal power splatters into adjacent channels.
  - Containing 99-percent of the signal power in the channel is arbitrary.
- There is a tradeoff in the choice of percent power.
  - Let $\psi < 1$ be the fraction of power in the band.
  - Then the fraction of power in each adjacent channel is $K_s = (1 - \psi)/2$.
- Each transmitting mobile has probability of an *adjacent-channel collision* given by

$$p_{\mathsf{a}} = D\left[\left(\frac{2}{L}\right)\left(\frac{L-2}{L}\right) + \left(\frac{1}{L}\right)\left(\frac{2}{L}\right)\right] = \frac{2D(L-1)}{L^2}.$$

SINR

The performance at the reference receiver is characterized by the *signal-to-interference and noise ratio* (SINR), given by:

$$\gamma = \frac{g_0 \Omega_0}{\Gamma^{-1} + \sum_{i=1}^{M} I_i g_i \Omega_i} \quad (1)$$

where:

- Γ is the SNR at unit distance.
- g_i is the power gain due to Nakagami fading.
- I_i is a discrete random variable that characterizes the type of collision:

$$I_i = \begin{cases} \psi & \text{with probability } p_{\text{c}} \text{ for co-channel collision} \\ K_s & \text{with probability } p_{\text{a}} \text{ for adjacent-channel collision} \\ 0 & \text{with probability } p_{\text{n}} = 1 - p_{\text{c}} - p_{\text{a}} \text{ for no collision} \end{cases} \quad (2)$$

- $\Omega_i = \frac{P_i}{P_0} 10^{\xi_i/10} ||X_i||^{-\alpha}$ is the normalized receiver power from transmitter i in presence of log-normal shadowing.
  - P_i is the power of transmitter i, assumed to be constant for $\forall i$.
  - α is the path loss.
  - $||X_0|| = 1$ to normalize distance.
  - ξ_i are i.i.d. zero-mean Gaussian with standard deviation σ_s dB.

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Outage Probability

- An *outage* occurs when the SINR is below a threshold β.
  - β depends on the choice of modulation and coding.
- The *outage probability* is

$$\epsilon = P\left[\gamma \leq \beta\right]. \quad (3)$$

- Substituting (1) into (3) and rearranging yields

$$\epsilon = P\Big[\underbrace{\beta^{-1} g_0 \Omega_0 - \sum_{i=1}^{M} I_i g_i \Omega_i}_{\mathsf{Z}} \leq \Gamma^{-1}\Big].$$

- The outage probability is related to the cdf of Z,

$$\epsilon = P\left[\mathsf{Z} \leq \Gamma^{-1}\right] = F_{\mathsf{Z}}(\Gamma^{-1}).$$

Outage Probability Derivation

The cdf conditioned
on the network geometry, $\mathbf{\Omega}$:
$$\epsilon_o = P\left[\gamma \leq \beta \middle| \mathbf{\Omega}\right] = F_{\mathsf{Z}}(\Gamma^{-1} \middle| \mathbf{\Omega})$$

Outage Probability Derivation

The cdf conditioned
on the network geometry, $\mathbf{\Omega}$:
$\epsilon_o = P\left[\gamma \leq \beta \middle| \mathbf{\Omega}\right] = F_{\mathsf{Z}}(\Gamma^{-1} | \mathbf{\Omega})$

The cdf averaged
over the spatial distribution:

$$\begin{aligned}\epsilon = \mathbb{E}\left[\epsilon_o\right] &= F_{\mathsf{Z}}(\Gamma^{-1}) \\ &= \int f_{\mathbf{\Omega}}(\boldsymbol{\omega}) F_{\mathsf{Z}}(\Gamma^{-1} | \boldsymbol{\omega}) d\boldsymbol{\omega} \end{aligned} \quad (4)$$

$$f_{\mathbf{\Omega}}(\boldsymbol{\omega}) = \prod_{i=1}^{M} f_{\Omega_i}(\omega_i)$$
is the pdf of $\mathbf{\Omega}$

Conditional Outage Probability

The outage probability conditioned on the network geometry:

$$\bar{F}_{\mathsf{Z}}\left(z \middle| \boldsymbol{\Omega}\right) = e^{-\beta_0 z} \sum_{j=0}^{m_0-1} (\beta_0 z)^j \sum_{k=0}^{j} \frac{z^{-k} H_k(\boldsymbol{\Omega})}{(j-k)!} \quad (5)$$

where $\beta_0 = m_0 \beta / \left(\Psi \Omega_0\right)$,

$$H_k(\boldsymbol{\Omega}) = \sum_{\substack{\ell_i \geq 0 \\ \sum_{i=0}^{M} \ell_i = k}} \prod_{i=1}^{M} G_{\ell_i}(\Omega_i), \quad (6)$$

the summation in (6) is over all sets of positive indices that sum to k, and

$$G_{\ell_i}(\Omega_i) = p_{\mathsf{n}} \delta_{\ell_i} + \frac{\Gamma(\ell_i + m_i)}{\ell_i! \Gamma(m_i)} \left[p_{\mathsf{c}} \phi_i(\psi) + p_{\mathsf{a}} \phi_i(K_s) \right],$$

δ_ℓ is the Kronecker delta function, and

$$\phi_i(x) = \left(\frac{x \Omega_i}{m_i} \right)^{\ell_i} \left(\frac{x \beta_0 \Omega_i}{m_i} + 1 \right)^{-(m_i + \ell_i)}.$$

Conditional Outage Probability

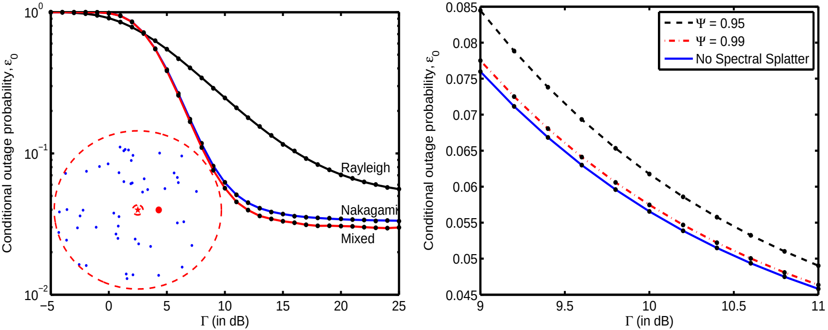
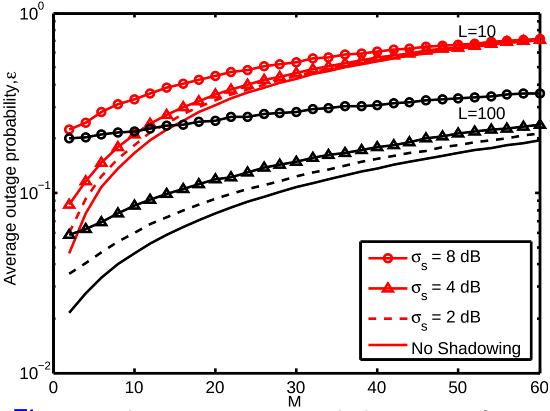


Figure: *Conditional outage probability as a function of SNR Γ.*

Example:

- $M = 50$ interferers.
- Annular network ($r_{ex} = 0.25$ and $r_{net} = 4$).
- $\alpha = 3$.
- $L = 200$ and $D = 1$.
- $\beta = 3.7$ dB.
- Analytical curves are the lines, while • represents simulated values.

Average Outage Probability



Example:

- $r_{ex} = 0.25$.
- $r_{net} = 4$.
- $\alpha = 3$.
- $\beta = 3.7$ dB.
- $D = 1$.
- $\Gamma = 10$ dB.
- No spectral splatter $(\Psi = 1)$.
- Mixed fading: $m_0 = 4$ and $m_i = 1$ for $i \geq 1$.

Figure: Average outage probability as a function of the number of interferers M for two values of L. For each L, curves are shown for the case of no shadowing, and for shadowing with three values of σ_s.

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Transmission Capacity: Definition

- The *transmission capacity* (TC) is the spatial spectral efficiency.
- If there are λ mobiles per unit area, then the number of successful transmissions per unit area is

$$\tau = \lambda(1-\epsilon)$$

- If the outage probability ϵ is constrained to not exceed ζ, then the transmission capacity is

$$\tau_c(\zeta) = \epsilon^{-1}(\zeta)(1-\zeta) \quad (7)$$

where $\epsilon^{-1}(\zeta)$ is the maximum mobile density such that $\epsilon \leq \zeta$.

Modulation-Constrained TC

- The *modulation-constrained* transmission capacity (MCTC) is

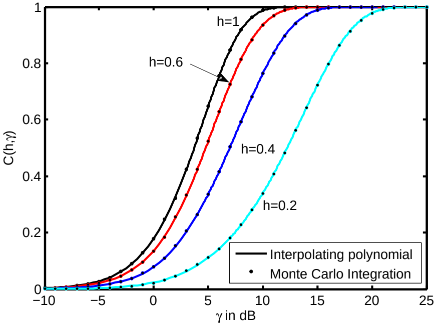
$$\tau' = \lambda(1-\epsilon)\left(\frac{RD\eta(h,\Psi)}{L}\right)$$

  where
  - R is the rate of the channel code.
  - h is the modulation index.
  - $\eta(h,\Psi)$ the modulation's spectral efficiency (bps/Hz).
  - $\epsilon = P[C(\gamma) \leq R] = P[\gamma \leq C^{-1}(R)]$, where $C(\gamma)$ is the modulation-constrained capacity with SNR γ.
  - L is the number of channels.
  - D is the duty factor.
  - λ is the density of mobiles.
  - τ' has units of $bps/Hz/m^2$.
- For a given λ, Γ, and spatial model, there is a set of (L, R, h, Ψ) that maximizes τ'.

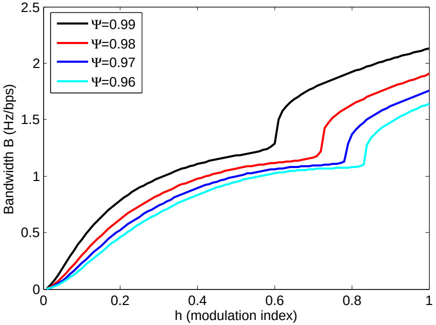
Noncoherent Binary CPFSK

FH systems often use noncoherent continuous-phase frequency-shift keying (CPFSK).

- The modulation-constrained capacity $C(h, \gamma)$ of binary CPFSK is shown as a function of γ in figure (a).
- The bandwidth $B = 1/\eta(h, \Psi)$ is shown as function of the modulation index h in figure (b).



(a) channel capacity versus SNR γ



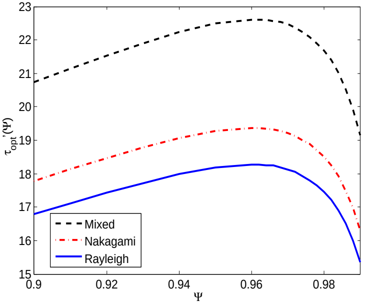
(b) bandwidth versus modulation index

[7] S. Cheng, R. Iyer Sehshadri, M.C. Valenti, and D. Torrieri, "The capacity of noncoherent continuous-phase frequency shift keying", in *Proc. Conf. on Info. Sci. and Sys. (CISS)*, (Baltimore, MD), Mar. 2007.

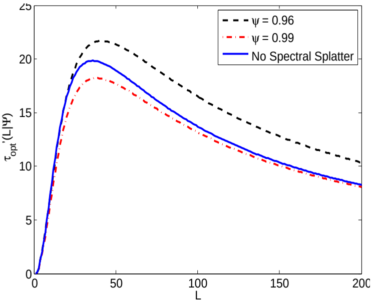
Outline

Optimization Objectives

- For a given network model (r_{ex}, r_{net}, M) and channel, that takes in account for shadowing (σ_s), path loss (α), Nakagami fading (m_0 and m_i) and channel SNR Γ, the values of (L, R, h, Ψ) that maximize the modulation-constrained transmission capacity τ' are found.
- Optimization is through a simplex-search strategy (Nelder-Mead method) since the optimization surface is convex.



(c) MCTC vs Ψ. At each value of Ψ, $\{L, R, h\}$ are varied to maximize TC.



(d) MCTC vs L for mixed fading.

Example:

- $M = 50.$
- $\alpha = 3.$
- $||X_0|| = 1.$
- $r_{net} = 2.$
- $r_{ex} = 0.25.$
- $\sigma_s = 8$ dB.
- $\Gamma = 10$ dB.
- $D = 1.$

Example of optimization

r_{net}	σ_s	Fading	L	R	h	ψ	τ'_{opt}
2	0	Rayleigh	36	0.64	0.81	0.96	17.74
		Nakagami	45	0.64	0.81	0.96	19.88
		Mixed	40	0.64	0.81	0.96	22.59
	8	Rayleigh	34	0.64	0.81	0.96	18.29
		Nakagami	44	0.66	0.81	0.96	19.36
		Mixed	38	0.64	0.81	0.96	22.09
4	0	Rayleigh	13	0.57	0.85	0.95	11.92
		Nakagami	16	0.54	0.85	0.95	13.23
		Mixed	15	0.56	0.85	0.95	14.64
	8	Rayleigh	12	0.58	0.85	0.95	12.22
		Nakagami	15	0.54	0.85	0.95	13.13
		Mixed	14	0.57	0.85	0.95	14.55

Table: *Results of the optimization for $M = 50$ interferers and $\Gamma = 10$ dB. The normalized MCTC τ' is in units of bps/kHz-m^2.*

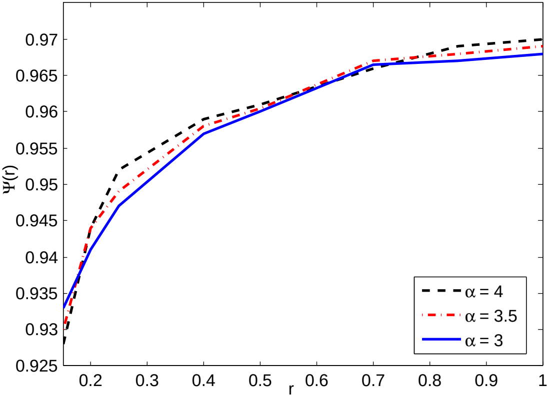
Outline

Conclusions

- The performance of frequency-hopping ad hoc networks is a function of:
  - Number of hopping channels L.
  - Code rate R.
  - Modulation index h (if CPFSK modulation).
  - Fractional in-band power Ψ.
- These parameters can be jointly optimized.
  - Transmission capacity is the objective function of choice.
  - The modulation-constrained TC quantifies the tradeoffs involved.
- The approach is general enough to handle a wide variety of conditions. Examples of future extensions:
  - Any spatial model, including repulsion models.
  - Adaptive code rates (R not fixed for all users).
- Additional constraints can be imposed on the optimization.
  - Per-node outage constraint.
  - Fixed or minimum data rates per user.

Thank You

Appendix: Effect of Normalized Distance



Example:

- $M = 50$.
- $r_{ex} = 0.25$.
- $r_{net} = 4$.
- $||X_0|| = 1$.
- $\sigma_s = 8$ dB.
- $D = 1$.
- $\Gamma = 10$ dB.
- Mixed fading: $m_0 = 4$ and $m_i = 1$ for $i \geq 1$.

Figure: Optimal fractional in-band power $\Psi(r)$ as function of normalized transmitter distance $r = |X_0|/r_{net}$. Results are shown for three different path-loss coefficients.

Appendix: Average Outage Probability

Recall: The averaged outage probability is found by

$$F_{\mathsf{Z}}(\Gamma^{-1}) = \int \left(\prod_{i=1}^{M} f_{\Omega_i}(\omega_i) \right) F_{\mathsf{Z}}(\Gamma^{-1} | \boldsymbol{\omega}) d\boldsymbol{\omega} \tag{8}$$

• In **absence of shadowing** $\Omega_i = c_i^{-1} ||X_i||^{-\alpha}$, which has pdf

$$f_{\Omega_i}(\omega_i) = \begin{cases} \dfrac{2c_i^{2/\alpha} \omega_i^{-\left(\frac{2+\alpha}{\alpha}\right)}}{\alpha(r_{net}^2 - r_{ex}^2)} & \text{for } c_i r_{net}^{-\alpha} \leq \omega_i \leq c_i r_{ex}^{-\alpha} \\ 0 & \text{elsewhere} \end{cases} \tag{9}$$

where $c_i = (P_i/P_0)$.

Appendix: Average Outage Probability

Substituting (9) and (5) into (8), the cdf of Z is found to be

$$\bar{F}_{\mathsf{Z}}(z) = e^{-\beta_0 z} \sum_{s=0}^{m_0-1} (\beta_0 z)^s \sum_{t=0}^{s} \frac{z^{-t}}{(s-t)!} \sum_{\substack{\ell_i \geq 0 \\ \sum_{i=0}^{M} \ell_i = t}} \prod_{i=1}^{M} \mathbb{E}_{\Omega}\left(\Omega_i\right) \tag{10}$$

where

$$\begin{aligned} \mathbb{E}_{\Omega}\left(\Omega_i\right) &= p_{\mathsf{n}} \delta_{\ell_i} + \frac{2 m_i^{m_i} \Gamma(\ell_i + m_i)}{\alpha(\ell_i!)\Gamma(m_i)\beta_0^{(m_i+\ell_i)}} \left\{ p_{\mathsf{c}} \left[I\left(\frac{\psi c_i}{r_{\mathsf{net}}^{\alpha}}\right) - I\left(\frac{\psi c_i}{r_{\mathsf{ex}}^{\alpha}}\right) \right] + \right. \\ & \left. p_{\mathsf{a}} \left[I\left(\frac{K_s c_i}{r_{\mathsf{net}}^{\alpha}}\right) - I\left(\frac{K_s c_i}{r_{\mathsf{ex}}^{\alpha}}\right) \right] \right\}, \end{aligned} \tag{11}$$

$c_i = (P_i/P_0)$ and ${}_2F_1$ is the Gauss hypergeometric function,

$$I(x) = \frac{{}_2F_1\left(\left[m_i + \ell_i, m_i + \frac{2}{\alpha}\right]; m_i + \frac{2}{\alpha} + 1; -\frac{m_i}{x\beta_0}\right)}{x^{m_i}\left(m_i + \frac{2}{\alpha}\right)} \tag{12}$$

and ${}_2F_1([a,b], c, z)$ is the Gauss hypergeometric function.

Appendix: Average Outage Probability

- In **presence of shadowing** $\Omega_i = c_i^{-1} 10^{\xi_i/10} ||X_i||^{-\alpha}$, which has pdf

$$f_{\Omega_i}(\omega_i) = \begin{cases} \omega^{-\frac{2+\alpha}{\alpha}} \dfrac{\left[\zeta\left(c_i \omega_i r_{\text{net}}^{\alpha}\right) - \zeta\left(c_i \omega_i r_{\text{ex}}^{\alpha}\right)\right]}{\alpha c_i^{2/\alpha}\left(r_{\text{net}}^2 - r_{\text{ex}}^2\right)} & \text{for } 0 \le \omega_i \le \infty \\ 0 & \text{elsewhere} \end{cases}$$

where

$$\zeta(z) = \text{erf}\left(\frac{\sigma_s^2 \ln^2(10) - 50\alpha \ln(z)}{5\sqrt{2}\alpha\sigma_s \ln(10)}\right) e^{\frac{\sigma_s^2 \ln^2(10)}{50\alpha^2}} \tag{13}$$

Because $||X_0||$ is deterministic, $\Omega_0 = 10^{\xi_0/10}||X_0||^{-\alpha}$ is a log-normal variable with pdf

$$f_{\Omega_0}(\omega) = \frac{10\left(2\pi\sigma_s^2\right)^{-\frac{1}{2}}}{\ln(10)\omega} \exp\left\{-\frac{10^2 \log_{10}^2\left(||X_0||^{\alpha}\omega\right)}{2\sigma_s^2}\right\} \tag{14}$$

for $0 \le \omega \le \infty$, and zero elsewhere.

Appendix: Average Outage Probability

Substituting (13), (14) and (5) into (8), the cdf of Z is found to be

$$\bar{F}_{Z_M}(z) = \sum_{s=0}^{m_0-1}\sum_{t=0}^{s}\frac{z^{-t}}{(s-t)!}\sum_{\substack{\ell_i\geq 0\\ \sum_{i=0}^{M}\ell_i=t}}\int_0^\infty \exp\left\{-\frac{\beta m_0 z}{\Psi y}\right\}\left(\frac{\beta m_0 z}{\Psi y}\right)^s \prod_{i=1}^{M}\left[p_n\delta_{\ell_i}+p_c\Phi_i(y,\psi)+p_a\Phi_i(y,K_s)\right]f_{\Omega_0}(y)dy \quad (15)$$

where

$$\Phi_i(y,\chi) = \frac{\Gamma(\ell_i+m_i)}{\ell_i!\Gamma(m_i)}\int_0^\infty f_{\Omega_i}(\omega)\left(\frac{\chi\omega}{m_i}\right)^{\ell_i}\left(\frac{\chi\beta m_0\omega}{\psi m_i y}+1\right)^{-(m_i+\ell_i)}d\omega. \quad (16)$$