# Interference and Spatial Modeling in Wireless Networks

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Collaborators:

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- Mr. Salvatore Talarico (WVU Ph.D. student).

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# Outline

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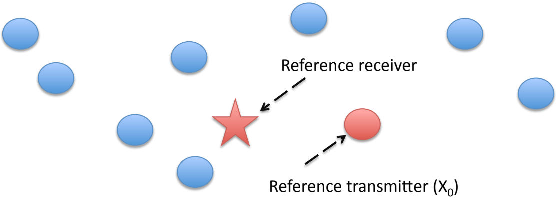
# Introduction

- Sources of randomness in a wireless network:
  1. Fading and noise.
  2. Random transmitter activity (buffer status).
  3. Effect of interference-avoidance protocols.
  4. Shadowing.
  5. Location of interfering transmitters (topology).
- The above effects work at different timescales.
  - Suggests a hierarchical approach to analysis.
  - Fix the locations of the interferers and amount of shadowing; determine the corresponding *conditional* outage probability.
  - Then consider the effect of node location and shadowing distribution.
- Averages are more appropriate for faster phenomena (fading).
  - Closed-form expressions can be obtained for the outage probability of a given network topology.
- For slower phenomena (network topology), cdf of performance metrics (outage, rate) are more meaningful.

# Outline

# Network Model




- Transmitters are arbitrarily placed in a 2-D finite region.
- A reference receiver is located at the origin.
- $X_i$ represents the $i^{th}$ transmitter and its location.
  - $X_0$ is location of the reference transmitter.
  - $M$ interfering transmitters, $\{X_1, ..., X_M\}$.
  - $|X_i|$ is distance from $i^{th}$ transmitter to the reference receiver.
- Network may be *cellular* or *ad hoc* (infrastructureless).
- Later, we will consider models governing the placement of the radios.

# Channel Model

- Signal from $X_i$ to the reference receiver undergoes fading, shadowing, and path loss.
- At the reference receiver, $X_i$'s signal is received with power:

$$\rho_i = P_i g_i 10^{\xi_i/10} \left( \frac{|X_i|}{d_0} \right)^{-\alpha}$$

where:

- $P_i$ is the transmitted power.
- $g_i$ is the power gain due to fading.
- $\xi_i$ is the dB shadowing factor.
- $\alpha$ is the path-loss exponent.
- $d_0$ is a reference distance.

# SINR

The performance at the reference receiver is characterized by the signal-to-interference and noise ratio (SINR), given by:

$$\gamma = \frac{\rho_0}{\mathcal{N} + \sum_{i=1}^{M} I_i \rho_i} = \frac{g_0 \Omega_0}{\Gamma^{-1} + \sum_{i=1}^{M} I_i g_i \Omega_i} \quad (1)$$

where:

- $\mathcal{N}$ is the noise power.
- $I_i$ is a Bernoulli indicator with $P[I_i] = p_i$.
  - Probability that $X_i$ collides with $X_0$.
- $\Gamma = P_0/\mathcal{N}$ is the SNR.
- $\Omega_i = \frac{P_i}{P_0} 10^{\xi_i/10} |X_i|^{-\alpha}$ is the normalized average received power.
  - Normalized with respect to $P_0$
  - Accounts for shadowing and path loss, but not fading.

# Outage Probability

- An *outage* occurs when $\gamma \leq \beta$, where $\beta$ is an SINR threshold.
  - The value of $\beta$ depends on the choice of modulation and coding.
- From (1), the outage probability is

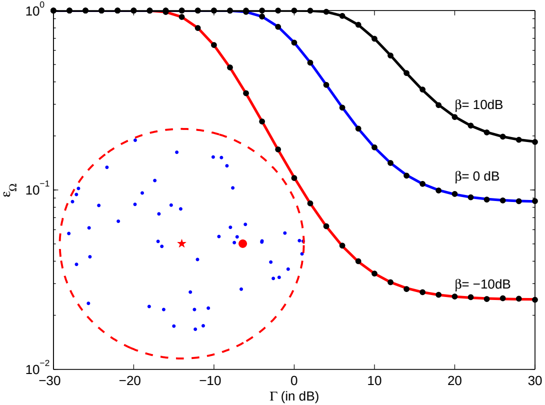
$$\begin{aligned}\epsilon &= P\Big[\underbrace{\tfrac{g_0\Omega_0}{\Gamma^{-1}+\sum_{i=1}^{M} I_i g_i \Omega_i}}_{\gamma} \leq \beta\Big] \\ &= P\Big[\underbrace{\beta^{-1} g_0 \Omega_0 - \sum_{i=1}^{M} I_i g_i \Omega_i}_{\mathsf{Z}} \leq \Gamma^{-1}\Big] \\ &= P\left[\mathsf{Z} \leq \Gamma^{-1}\right] = F_{\mathsf{Z}}(\Gamma^{-1}).\end{aligned}$$

- The outage probability is the cdf of Z, $F_{\mathsf{Z}}(z)$ evaluated at $z = \Gamma^{-1}$.
  - Since $\{\Omega_i\}$ are fixed, we can also write as the *conditional* cdf $F_{\mathsf{Z}}(z \big| \boldsymbol{\Omega})$.

# Rayleigh Fading

- When the $g_i$ are i.i.d. exponential, the outage probability conditioned on the network geometry is:

$$F_Z(z|\boldsymbol{\Omega}) = 1 - e^{-\beta\Omega_0^{-1}z}\prod_{i=1}^{M}\left[\frac{(1-p_i)\beta\Omega_0^{-1}+\Omega_i^{-1}}{\beta\Omega_0^{-1}+\Omega_i^{-1}}\right] \quad (2)$$




Example #1:

- $X_0 = 1$.
- $M = 50$ interferers, with $0.25 \le |X_i| \le 2, \forall i > 0$.
- $\alpha = 3$ and no shadowing.
- $p_i = 0.005$.
- Analytical curves are solid, while • represents simulated values.

# Nakagami Fading[1]

If the amplitude gain from the $i^{th}$ transmitter to the receiver is Nakagami-m with parameter $m_i$, then the outage probability conditioned on the network geometry is:

$$F_{\mathsf{Z}}\left(z \middle| \boldsymbol{\Omega}\right) = 1 - e^{-\beta_0 z} \sum_{j=0}^{m_0-1} (\beta_0 z)^j \sum_{k=0}^{j} \frac{z^{-k}}{(j-k)!} \sum_{\substack{\ell_i \geq 0 \\ \sum_{i=0}^{M} \ell_i = k}} \prod_{i=1}^{M} G_{\ell_i}[i] \quad (3)$$

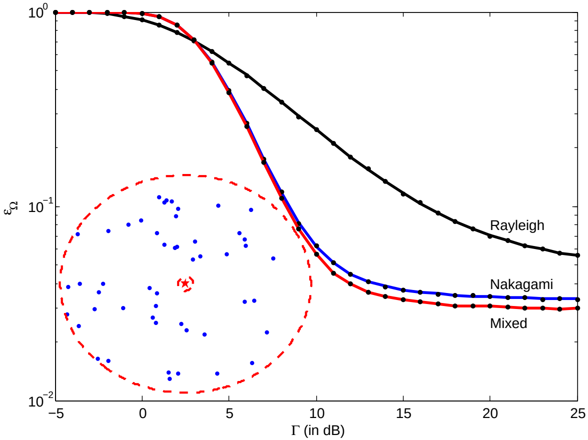
where $m_0$ is an integer, $\beta_0 = m_0 \beta / \Omega_0$, the summation in (3) is over all sets of positive indices that sum to $k$,

$$G_{\ell}[i] = (1-p_i)\delta[\ell] + \frac{p_i \Gamma(\ell + m_i)}{\ell! \Gamma(m_i)} \left(\frac{\Omega_i}{m_i}\right)^{\ell} \left(\frac{m_i}{\beta_0 \Omega_i + m_i}\right)^{m_i+\ell}, \quad (4)$$

and $\delta[\ell]$ is the Kroneker delta function.

[1] D. Torrieri and M.C. Valenti, "The outage probability of a finite ad hoc network in Nakagami fading," *IEEE T. Comm.*, Nov. 2012.

# Nakagami Fading Example




Example #2:

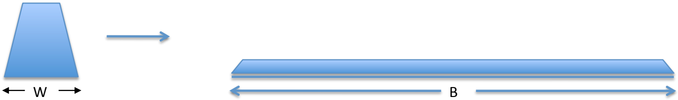
- $X_0 = 1$.
- $M = 50$ interferers, with $0.25 \leq |X_i| \leq 4, \forall i > 0$.
- $\alpha = 3$ and no shadowing.
- $p_i = 0.005$.
- $\beta = 3.7$ dB.
- Fading types:
  - Rayleigh: $m_i = 1, \forall i$.
  - Nakagami: $m_i = 4, \forall i$.
  - Mixed: $m_0 = 4$, $m_i = 1, \forall i > 0$.

Figure: Outage probability $\epsilon_\Omega$ conditioned on $\mathbf{\Omega}$ as a function of SNR $\Gamma$. Analytical curves are solid, while • represents simulated values.

# Interference Reduction

- Interference may be controlled through signal and protocol design.
- Spread spectrum signaling reduces interference.
  - Direct sequence (DS).
  - Frequency hopping (FH).
  - Hybrid DS/FH.
- Interference avoidance protocols reduce interference.
  - CSMA/CA.

# Direct Sequence Spread Spectrum

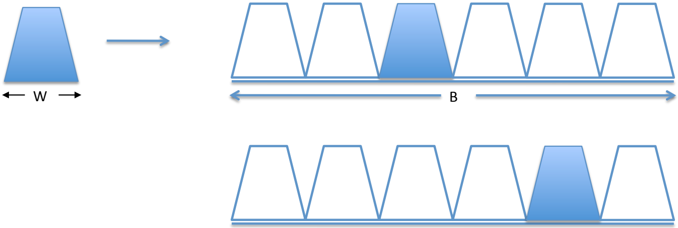



- Spread bandwidth of each signal by a *spreading factor* $G = B/W$.
- Effectively reduces the power of each interferer.
- $G$ is also called the *processing gain* and is the amount of reduction in interference power.
- Can be handled by dividing normalized powers of the interferers by $G$

$$\Omega_i = \left(\frac{1}{G}\right)\frac{P_i}{P_0}10^{\xi_i/10}|X_i|^{-\alpha}, \forall i > 0$$

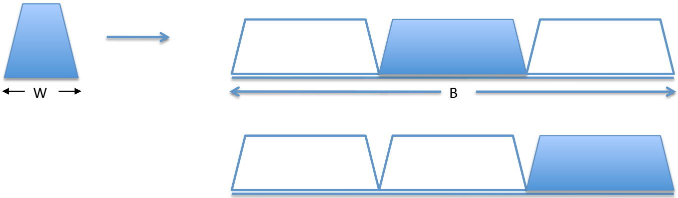
- Interference *averaging*.
- If transmissions are asynchronous a *chip factor* can further reduce the interference power.

# Frequency Hopping



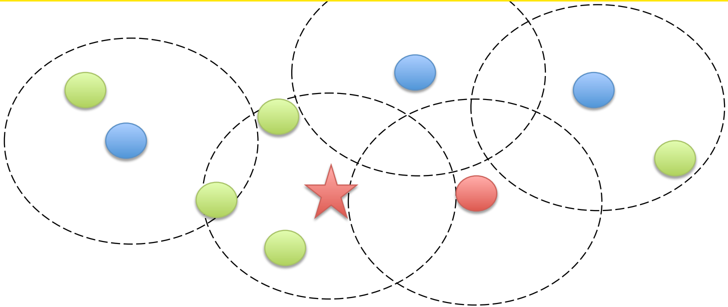

- Transmitters randomly pick from among $L$ frequencies.
- $I_i$ is a Bernoulli random variable with probability $p_i = 1/L$.
- Interference *avoidance*.

# Hybrid DS/FH




- Spread bandwidth of each signal by a factor $G$.
- Transmit DS-spread signal over randomly selected frequency.
- $G > 1$ and $p_i < 1$.

# Guard Zones[2]



- Interference-avoidance protocols may be used to prevent close interferers.
  - Carrier-sense multiple access with collision avoidance (CSMA-CA).
- If one transmitter is too close to another, it will deactivate.
  - Each transmitter is surrounded by a circular *guard zone* of radius $r_{\text{min}}$.
  - Other nodes in the guard zone are forbidden to transmit ($p_i = 0$).
- Equivalent to *Matern thinning* of the spatial model.

[2] D. Torrieri and M.C. Valenti, "Exclusion and guard zones in DS-CDMA ad hoc networks," *IEEE T. Comm.*, to appear.

# Guard Zones Reduce the Outage Probability

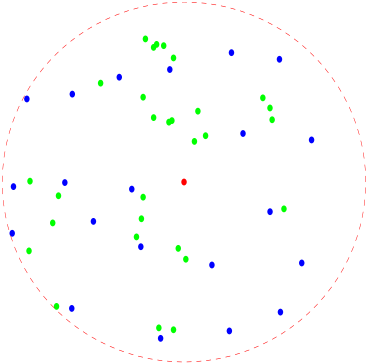


Figure : By using a guard zone with $r_{\text{min}} = 1$, the number of potential interferers decreases to 21.

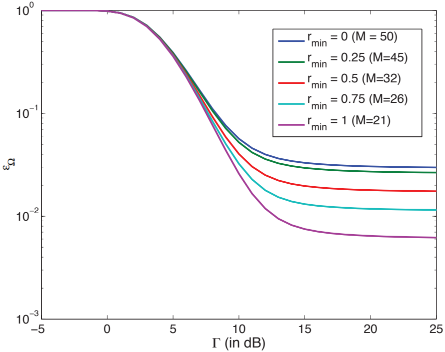



Figure : Outage probability $\epsilon$ over the mixed fading channel of Example #2 when a guard zone of radius $r_{\text{min}}$ is imposed.

# Outline

# Effect of Spatial Model

- So far, the outage probability has been conditioned on the *network geometry*.
  - The locations of potential interferers and the shadowing realization are fixed.
  - Only the fading and node activity are random.
- May want to remove the conditioning on network geometry.
  - Allow the transmitter locations and shadowing factors to randomly vary.
  - Particularly important for considering guard zones, which cause the locations of potential interferers to change.
- Should take into account the spatial model, which is a point process.
  - Binary point process (BPP).
  - Poisson point process (PPP).
  - Uniform clustering model.
  - Matern thinning.

# Spatially Averaged Outage Probability

The cdf conditioned
on the network geometry, $\mathbf{\Omega}$:
$$\epsilon_\Omega = P\left[\gamma \leq \beta \middle| \mathbf{\Omega}\right] = F_{\mathsf{Z}}(\Gamma^{-1} \big| \mathbf{\Omega})$$

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$$\epsilon_\Omega = P\left[\gamma \leq \beta \middle| \mathbf{\Omega}\right] = F_{\mathsf{Z}}(\Gamma^{-1} \middle| \mathbf{\Omega})$$

The cdf conditioned
on the number of interferers, $M$:
$$\epsilon_M = \mathbb{E}\left[\epsilon_\Omega\right] =$$
$$F_{\mathsf{Z}_M}(\Gamma^{-1}) = \int f_{\mathbf{\Omega}}(\boldsymbol{\omega}) F_{\mathsf{Z}}(\Gamma^{-1} \middle| \boldsymbol{\omega}) d\boldsymbol{\omega}$$

$$f_{\mathbf{\Omega}}(\boldsymbol{\omega}) = \prod_{i=1}^{M} f_{\Omega_i}(\omega_i)$$
is the pdf of $\mathbf{\Omega}$

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$$f_{\mathbf{\Omega}}(\boldsymbol{\omega}) = \prod_{i=1}^{M} f_{\Omega_i}(\omega_i)$$
is the pdf of $\mathbf{\Omega}$

The unconditional cdf:
$$\epsilon = \mathbb{E}\left[\epsilon_M\right] =$$
$$F_{\mathsf{Z}}(\Gamma^{-1}) = \sum_{m=0}^{\infty} p_M(m) F_{\mathsf{Z}_m}(z)$$

$p_M(m)$
is the pmf of M

# Average Outage Probability for a BPP: Results

- In a binomial point process (BPP), a fixed number of interferers are independently placed according to a uniform distribution.
- The cdf with a BPP network with interferers uniformly distributed on an annulus with inner radius $r_{\text{ex}}$ and outer radius $r_{\text{net}}$ is:

$$F_{\mathsf{Z}_M}(z) = \int f_{\boldsymbol{\Omega}}(\boldsymbol{\omega}) F_{\mathsf{Z}}(z \big| \boldsymbol{\omega}) d\boldsymbol{\omega} \quad (5)$$

where

$$f_{\Omega_i}(\omega) = \frac{2\omega^{\frac{2-\alpha}{\alpha}}}{\alpha\left(r_{\text{net}}^2 - r_{\text{ex}}^2\right)} \quad \text{for } r_{\text{ex}}^\alpha \leq \omega \leq r_{\text{net}}^\alpha.$$

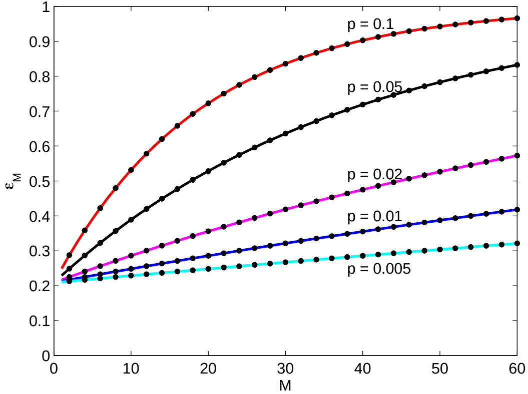
Substituting (2) into (5), the cdf of $\mathsf{Z}_M$ in Rayleigh fading is

$$F_{\mathsf{Z}_M}(z) = 1 - \exp\left\{-\beta\Omega_0^{-1} z\right\} \left[\frac{\Psi\left(r_{\text{net}}^\alpha\right) - \Psi\left(r_{\text{ex}}^\alpha\right)}{r_{\text{net}}^2 - r_{\text{ex}}^2}\right]^M \quad (6)$$

where $p_i = p, \forall i$, and

$$\Psi(x) = x^{\frac{2}{\alpha}} \left[1 - p + \frac{2p}{\alpha+2} \cdot \frac{x^{\frac{2+\alpha}{\alpha}}}{\beta\Omega_0^{-1}} \cdot {}_2F_1\left(\left[1, \frac{\alpha+2}{\alpha}\right]; \frac{2\alpha+2}{\alpha}, -\frac{x}{\beta\Omega_0^{-1}}\right)\right].$$

# Average Outage Probability for a BPP: Example




Example:

- $r_{\text{ex}} = 0.25$.
- $r_{\text{net}} = 2$.
- $\alpha = 3$.
- $\beta = 3.7$ dB.
- $\Gamma = 10$ dB.

Figure: Outage probability $\epsilon_M$ as a function of $M$ for five values of $p$ when the location of the nodes is drawn from a BPP. Analytical curves are solid, while the • were generated by randomly placing interferers and averaging the resulting conditional outage probabilities.

# Average Outage Probability for a PPP: Results

- In a Poisson point process (PPP), the number of interferers $M$ is a Poisson variable.
- The cdf with a PPP network is:

$$F_{\mathsf{Z}}(z) = \sum_{m=0}^{\infty} p_M(m) F_{\mathsf{Z}_m}(z) = \sum_{m=0}^{\infty} \frac{(\lambda A)^m}{m!} e^{-\lambda A} F_{\mathsf{Z}_m}(z) \quad (7)$$

where

  - $\lambda$ is the density of the interferers per unit area;
  - $A = \pi(r_{\text{net}}^2 - r_{\text{ex}}^2)$ is the area of the network;

substituting (6) into (7), the average outage probability is:

$$F_{\mathsf{Z}}(z) = 1 - e^{-\beta\Omega_0^{-1} z} e^{-\pi\lambda \cdot \left\{ r_{\text{net}}^2 - r_{\text{ex}}^2 - [\Psi(r_{\text{net}}^{\alpha}) - \Psi(r_{\text{ex}}^{\alpha})] \right\}} \quad (8)$$

- When $r_{\text{net}} \to \infty$, $r_{\text{ex}} = 0$, and $p = 1$:

$$F_{\mathsf{Z}}(z) = 1 - e^{-\beta_0 z} e^{\frac{-2\pi\lambda}{\alpha} \beta_0^{\frac{2}{\alpha}} \Gamma\left(\frac{2}{\alpha}\right) \Gamma\left(1 - \frac{2}{\alpha}\right)}$$

  - Baccelli et al. obtained same expression by using stochastic geometry.

# Average Outage Probability for a PPP: Examples[3]

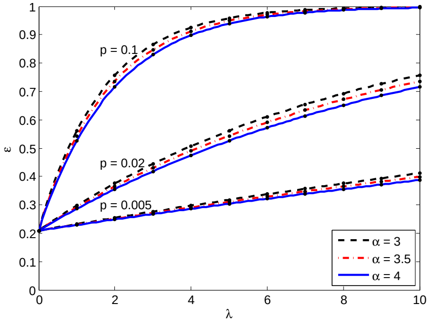



Figure : Parameters: SINR threshold $\beta = 3.7$ dB, SNR $\Gamma = 10$, inner radius $r_{\text{ex}} = 0.25$, and outer radius $r_{\text{net}} = 2$.

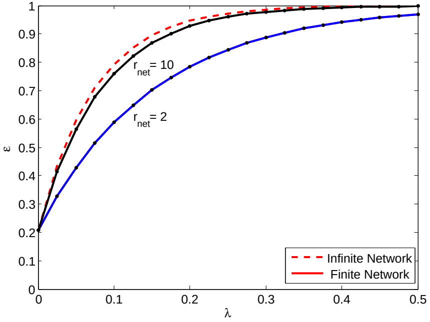



Figure : Parameters: SINR threshold $\beta = 3.7$ dB, SNR $\Gamma = 10$, inner radius $r_{\text{ex}} = 0$, $\alpha = 3$, and $p = 1$.

[3] S. Talarico, M.C. Valenti, and D. Torrieri, "Analysis and optimization of a frequency hopping ad hoc network in Rayleigh fading," in *Proc. Virginia Tech Symp. on Wireless Personal Commun.*, (Blacksburg, VA), June 2012

# Outline

# Transmission Capacity: Definition

- Reducing $\lambda$ reduces $\epsilon$, which improves the per-link throughput.
  - However, fewer links are supported, so less total data might be transmitted within the network.
- *Transmission capacity* is a metric that quantifies this tradeoff.
- If there are $\lambda$ transmitters per unit area, then the number of successful transmissions per unit area is

$$\tau = \lambda(1 - \epsilon)$$

- If the outage probability $\epsilon$ is constrained to not exceed $\zeta$, then the transmission capacity is

$$\tau_c(\zeta) = \epsilon^{-1}(\zeta)(1 - \zeta) \tag{9}$$

where $\epsilon^{-1}(\zeta)$ is the maximum mobile density such that $\epsilon \leq \zeta$.

# Transmission Capacity: Rayleigh Fading Results

- For the BPP case, $\epsilon^{-1}(\zeta)$ is found by solving $\epsilon = F_{\mathsf{Z}_M}(\Gamma^{-1}) = \zeta$ for $\lambda = M/A$ and then substituting into (9):

$$\tau_c(\zeta) = \frac{(1-\zeta)\left[\log\left(1-\zeta\right) + \beta\Omega_0^{-1}\Gamma^{-1}\right]}{A\log\left\{\left(r_{\text{net}}^2 - r_{\text{ex}}^2\right)^{-1}\left[\Psi\left(r_{\text{net}}^{\alpha}\right) - \Psi\left(r_{\text{ex}}^{\alpha}\right)\right]\right\}}$$

- In the PPP case, $\epsilon^{-1}(\zeta)$ is found by solving $\epsilon = F_{\mathsf{Z}}(\Gamma^{-1}) = \zeta$ for $\lambda$ and then substituting into (9):

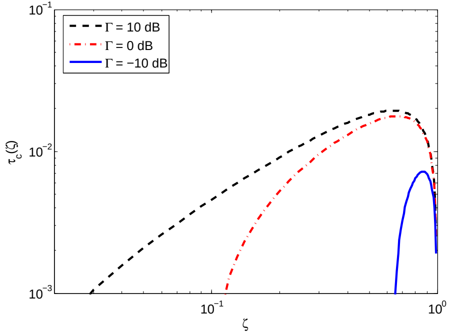
$$\tau_c(\zeta) = \frac{(1-\zeta)\left[\log\left(1-\zeta\right)^{-1} - \beta\Omega_0^{-1}\Gamma^{-1}\right]}{\pi\left\{r_{\text{net}}^2 - r_{\text{ex}}^2 - \left[\Psi\left(r_{\text{net}}^{\alpha}\right) - \Psi\left(r_{\text{ex}}^{\alpha}\right)\right]\right\}}.$$

- Asymptotically for a PPP, with $r_{\text{ex}} = 0$ and $p = 1$, as $r_{\text{net}} \to \infty$:

$$\tau_c(\zeta) = \frac{(1-\zeta)\left[\log\left(1-\zeta\right)^{-1} - \beta\Omega_0^{-1}\Gamma^{-1}\right]}{\pi\beta_0^{\frac{2}{\alpha}}\frac{2\pi}{\alpha}\csc\left(\frac{2\pi}{\alpha}\right)}$$

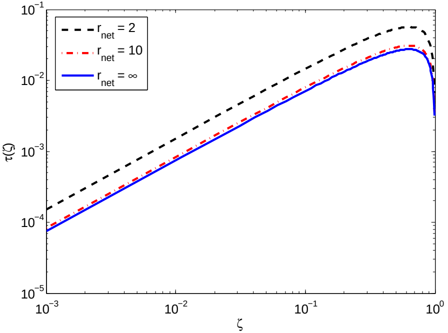
  - Weber et al. obtained the same expression using stochastic geometry.

# Transmission Capacity: Examples




Parameters:

- BPP
- $r_{\text{ex}} = 0$;
- $r_{\text{net}} = 2$;
- $p = 1$.
- $\beta = -10$ dB;
- $\alpha = 3$;




Parameters:

- PPP
- $r_{\text{ex}} = 0$;
- $\Gamma = 10$ dB.
- $p = 1$;
- $\beta = -10$ dB;
- $\alpha = 3$;

# Accounting for Modulation

- Until now, we have picked the SINR threshold $\beta$ arbitrarily.
- However, $\beta$ depends on the choice of modulation.
  - For *ideal* (input is a complex Gaussian) signaling

$$C(\gamma) = \log_2(1+\gamma)$$

    $\beta$ is the value of $\gamma$ for which $C(\gamma) = R$ (the code rate),

$$\beta = 2^R - 1$$

  - For other modulations, the *modulation-constrained* capacity $C(\gamma)$ must be used, which can be found by measuring the mutual information between channel input and output.

# Area Spectral Efficiency

- The *area spectral efficiency* accounts for not only the number of transmissions per unit area, but also the *rate* of the transmissions.
- Define as

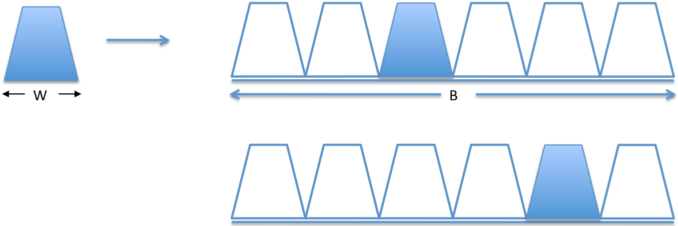
$$\tau' = R\eta p\lambda(1-\epsilon)$$

where

  - $R$ is the rate of transmission, which is related to $\beta$.
    - Units of (information) bits per channel symbol.
  - $\eta$ is the spectral efficiency of the modulation.
    - Units of channel symbols per Hz.
  - $\tau'$ has units of $bps/Hz/m^2$.

# Outline

# Frequency-Hopping Networks

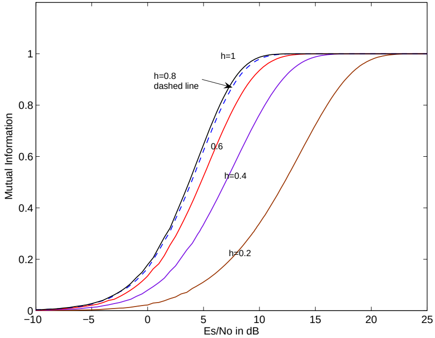



- The performance of a FH network depends on:
  - The number of hopping channels $L = B/W$.
  - The code rate $R$.
  - The modulation format, and its spectral efficiency $\eta$.
- For a fixed bandwidth $B$, there is a tradeoff among the above parameters.
- The network can be optimized by finding the parameters that maximize the area spectral efficiency[4].

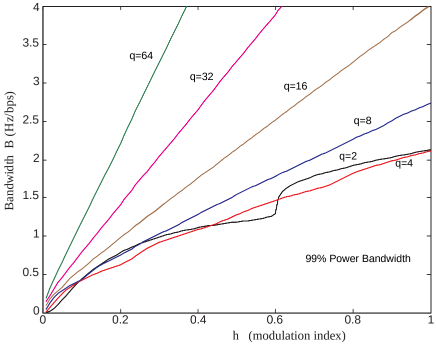
[4]M.C. Valenti, D. Torrieri, and S. Talarico, "Optimization of a finite frequency-hopping ad hoc network in Nakagami fading," *MILCOM*, 2012.

# Noncoherent Binary CPFSK

FH systems often use noncoherent continuous-phase frequency-shift keying, whose capacity & bandwidth depends on the modulation index $h$.



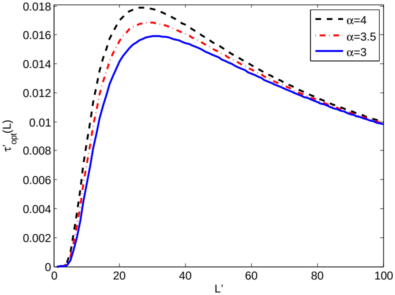

(a) channel capacity versus $\mathcal{E}_S/N_0$

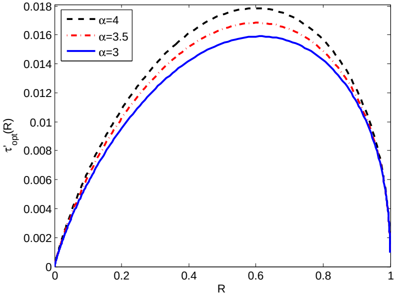


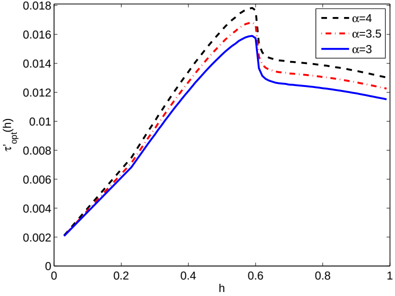

(b) bandwidth versus modulation index

[9] S. Cheng, R. Iyer Sehshadri, M.C. Valenti, and D. Torrieri, "The capacity of noncoherent continuous-phase frequency shift keying", in *Proc. Conf. on Info. Sci. and Sys. (CISS)*, (Baltimore, MD), Mar. 2007.

# Influence of Parameters: BPP



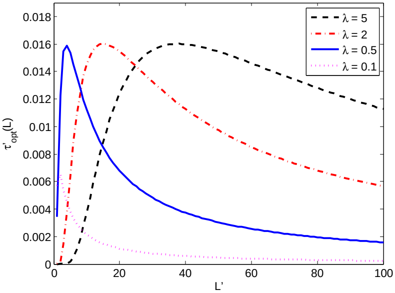



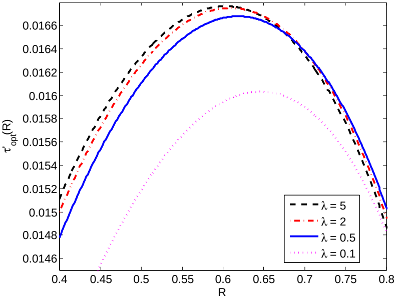



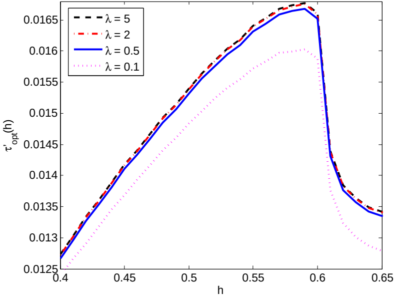

Parameters:

- $r_{\text{ex}} = 0.25$;
- $r_{\text{net}} = 2$;
- $\Gamma = 10$ dB.
- $M = 50$.
- Rayleigh fading.

# Influence of Parameters: PPP




Parameters:

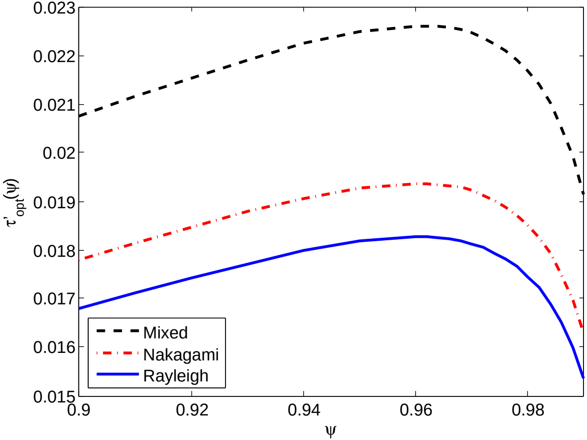
- $r_{\text{ex}} = 0.25$;
- $r_{\text{net}} = 2$;
- $\Gamma = 10$ dB.
- $\alpha = 3$.
- Rayleigh fading.

# Adjacent-Channel Interference[5]

- We assumed that all interference is co-channel interference.
  - However, FSK modulation is not completely bandlimited.
  - Some of the spectrum splatters into adjacent channels.
- Typically, the frequency channels are matched to the 99-percent bandwidth of the moduation.
  - The percent of the signal power splatters into adjacent channels.
  - Containing 99-percent of the signal power in the channel is arbitrary.
- There is a tradeoff in the choice of percent power.
  - Let $\psi < 1$ be the fraction of power in the band.
  - Then the fraction of power in each adjacent channel is $(1 - \psi)/2$.
- Can determine $\psi$ that maximizes the area spectral efficiency.
  - Redefine $I_i$ to be the fraction of interfering power in the channel.
  - $I_i = \psi$ w.p. $p_i = 1/L$, and $I_i = (1 - \psi)/2$ w.p. $p_i = \frac{2(L-1)}{L^2}$.

[5] M.C. Valenti, D. Torrieri, and S. Talarico, "Adjacent-channel interference in frequency-hopping ad hoc networks," *ICC*, 2013.

# ACI Optimization Example




Parameters:

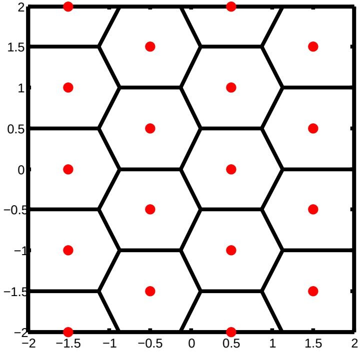
- $X_0 = 1$.
- $M = 50$ interferers, with $0.25 \leq |X_i| \leq 2, \forall i > 0$.
- $\alpha = 3$ and shadow std. $\sigma = 8$ dB.
- Fading types:
  - Rayleigh: $m_i = 1, \forall i$.
  - Nakagami: $m_i = 4, \forall i$.
  - Mixed: $m_0 = 4$, $m_i = 1, \forall i > 0$.

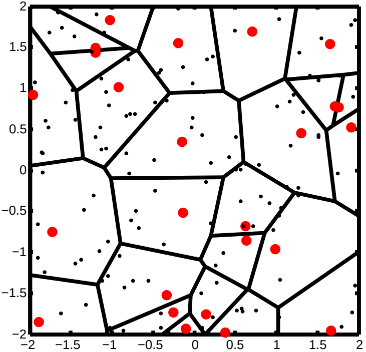
Figure: Maximum achievable area spectral efficiency $\tau'_{opt}(\psi)$ as a function of the fractional in-band power $\psi$. For each value of $\psi$, the modulation-index $h$, the number of frequency channels $L$, and code rate $R$ are varied to maximize the TC.

# Outline

# Extreme Cellular Network Models

- Current approaches form modeling cellular networks use one of two extremes:




Classic approach (regular grid):

- The analysis often focuses on the worst case-locations (cell edge).

Modern approach (stochastic geometry):

- Assumes infinite network;
- Base stations are drawn from a random point process
- No minimum separation.

# A Constrained Spatial Model

The minimum spacing between base stations is $r_{\text{bs}}$.

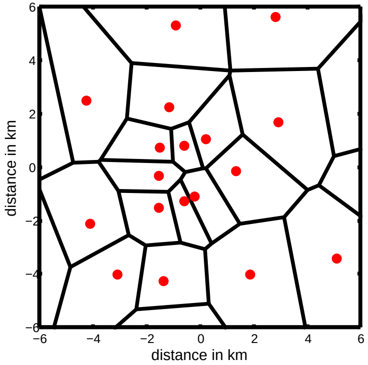



Figure : Actual base-station locations from a current cellular deployment in a small city with a hilly terrain.

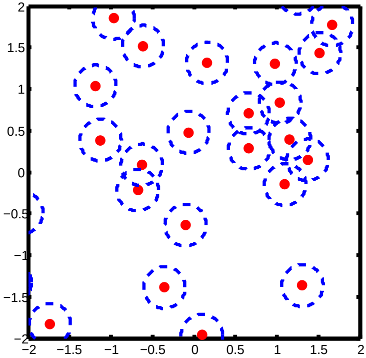



Figure : Simulated base-station locations when the minimum base-station separation is $r_{\text{bs}} = 0.25$.

# A Constrained Spatial Model

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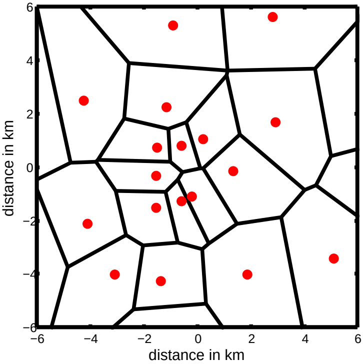


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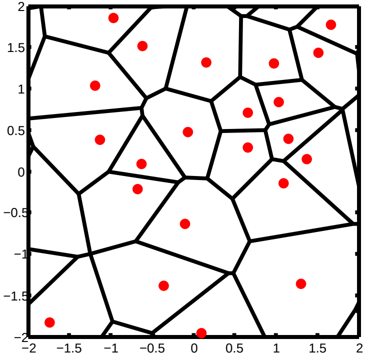


Figure : Simulated base-station locations when the minimum base-station separation is $r_{\text{bs}} = 0.25$. Cell boundaries are indicated.

# A Constrained Spatial Model

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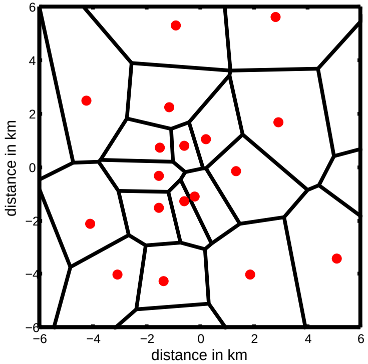


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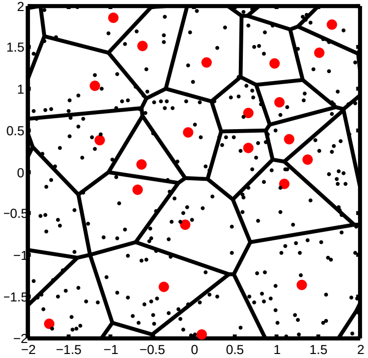
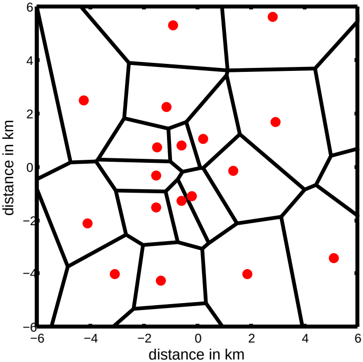



Figure : Simulated base-station locations when the minimum base-station separation is $r_{\text{bs}} = 0.25$. Cell boundaries are indicated, and the average cell load is

# A Constrained Spatial Model

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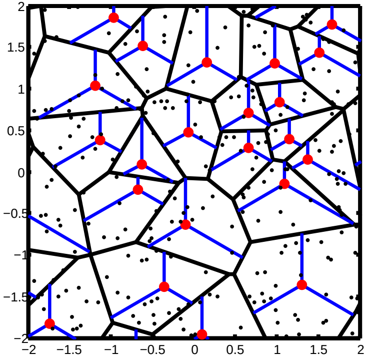


Figure : Simulated base-station locations when the minimum base-station separation is $r_{\text{bs}} = 0.25$. Cell and sector boundaries are indicated, and the average cell load is $16$ mobiles.

# Analysis of Cellular Networks

- Model can be used to analyze the downlink[6] or the uplink[7]
- Can be used to optimize resource allocation policies:
  - Power control.
  - Rate control.
  - Cell association.
- Fairness can be quantified by cdf of rate.
- The value of $r_{\text{bs}}$ can be obtained through statistical analysis of actual base station placements.
- Can use the analysis to develop Cooperative Multipoint (COMP) and Intercell Interference Coordination (ICIC) strategies.

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[6] M.C. Valenti, D. Torrieri, and S. Talarico, "A new analysis of the DS-CDMA cellular downlink under spatial constraints," *ICNC*, 2013.

[7] D. Torrieri, M.C. Valenti, and S. Talarico, "A new analysis of the DS-CDMA cellular uplink under spatial constraints," *ICC*, 2013.

# Downlink Performance

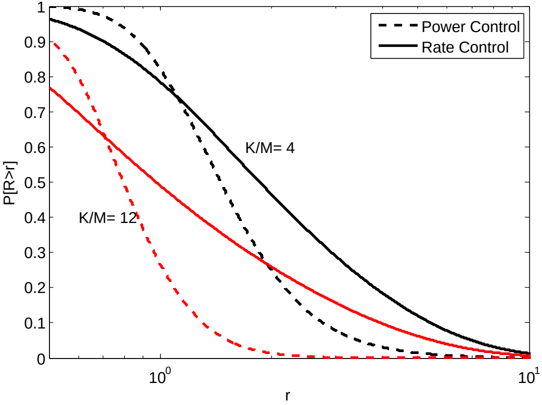


Figure : Complementary cdf of $R$ with either rate control or power control for a lightly loaded system $(K/M = 4)$ and a moderately-loaded system $(K/M = 12)$.

Example:

- $M = 50$ base stations;
- $K$ mobile users;
- $r_{net} = 2$;
- $r_{bs} = 0.25$;
- $\alpha = 3$;
- $\Gamma = 10$ dB;
- Spreading factor $G = 16$;
- Outage constraint $\hat{\epsilon} = 0.1$;
- Mixed fading:
  - $m = 3$ inside the cell.
  - $m = 1$ outside the cell.
- Shadow std. $\sigma_s = 8$ dB.

# Outline

# Conclusions

- The outage probability for any particular network realization can be found in closed form.
  - Captures the effects of fading, noise, and node activity.
  - No need to simulate the fading coefficients.
- Network topology remains as a source of randomness.
  - Locations of the transmitters.
  - Realization of shadowing.
  - Node deactivation due to interference-avoidance protocols.
- Analysis should focus on effects of topology.
  - Closed-form expressions for spatially averaged outage probability only possible for certain simple scenarios.
  - For more realistic spatial models, network topology needs to be simulated.
  - The cdf of rate is more useful than the average rate.
- Analysis can be used to study many problems related to ad hoc and cellular networks, including multihop routing.

# Thank you