Some New Adaptive Protocols for the Wireless Relay Channel

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- System constraints
  - "Cheap" and "cheaper" relay networks = TDD and TDM modes.
- Protocol alternatives
  - Nonadaptive: Always relay.
  - Source adaptive: During second time slot, source or relay transmits.
  - Relay adaptive: Relay switches between decode-forward and amplify-forward.
- Performance analysis
  - Capacity in AWGN.
  - Outage event probability in block fading.
- Conclusions

Cheap and Cheaper Relay Networks

- Unconstrained relay channel (van der Muelen; Cover & El Gamal)
  - Relay may simultaneously receive and transmit.
  - Source and relay transmit coherently (perfect transmit CSI): Beamforming.
  - Relay *networks*: Generalization to multiple relays (Gastpar & Vetterli).
- *Cheap* relay networks (Khojastepour & Aazhang; Høst Madsen)
  - Relay cannot receive and transmit simultaneously: TDD mode.
  - Source and relay may still transmit coherently: Beamforming.
- *Cheaper* relay networks (c.f. Laneman; Kramer, Gastpar & Gupta)
  - TDD relay.
  - Source and relay may not transmit coherently due to phase fading.
  - Orthogonal transmissions, e.g. TDM mode.
  - Besides being more feasible technically, the analysis is greatly simplified.

TDM Relay Networks

- General strategy:
  - Source creates a rate r codeword of length N.
  - Source begins by transmitting a fraction α of code symbols during the first time slot s_1.
  - During time slot s_2 either the source or relay may transmit the remaining fraction $\bar{\alpha} = 1 - \alpha$ of the codeword, depending on the protocol.
- Coding alternatives:
  - Repetition coding:
    - An identical rate $2r$ code is transmitted during each slot.
    - $\alpha = 1/2$.
    - Relay may simply amplify-and-forward (AF) or may decode, re-encode, and forward (DF).
    - Diversity-combining at destination (MRC).
    - SNRs add at the destination: *SNR accumulation.*
  - Incremental redundancy:
    - Through puncturing, different symbols of a rate r mother code are transmitted during each slot.
    - $0 < \alpha \leq 1$.
    - Relay must decode, re-encode, and transmit the remaining portion of codeword (DF).
    - Code-combining at destination (CC).
    - Capacities add at the destination: *Information accumulation.*

Outage Event Probability

- Block fading model:
  - SNR fixed for a codeword.
  - SNR varies independently from codeword-to-codeword.
- SNR definitions:
  - Instantaneous source-relay SNR: $\gamma_{s,r}$
  - Source-destination: $\gamma_{s,d}$
  - Relay-destination: $\gamma_{r,d}$
  - Average SNR: $\Gamma = E[\gamma]$
- Outage event probability (OEP)

$$\begin{aligned} P_o &= \int\int\int_{\mathcal{A}} p(\gamma_{s,r}, \gamma_{s,d}, \gamma_{r,d}) d\gamma_{s,r} d\gamma_{s,d} d\gamma_{r,d} \\ &= \frac{1}{\Gamma_{s,r}, \Gamma_{s,d}, \Gamma_{r,d}} \int\int\int_{\mathcal{A}} \exp\left\{ \left(\frac{\gamma_{s,r}}{\Gamma_{s,r}} + \frac{\gamma_{s,d}}{\Gamma_{s,d}} + \frac{\gamma_{r,d}}{\Gamma_{r,d}} \right) d\gamma_{s,r} d\gamma_{s,d} d\gamma_{r,d} \right\} \end{aligned} \quad (1)$$

  - $\mathcal{A} = \{\gamma_{s,r}, \gamma_{s,d}, \gamma_{r,d} : C < r\}$ is the outage event region.
  - C is the AWGN capacity of the protocol.

Decode-Forward Relaying with MRC

- Capacity:

$$C^*_{MRC} = \begin{cases} \min\left\{\frac{1}{2}C(\gamma_{s,d}+\gamma_{r,d}), \frac{1}{2}C(\gamma_{s,r})\right\} & : \quad \gamma_{s,d} < \gamma_{s,r} \\ \frac{1}{2}C(\gamma_{s,d}) & : \quad otherwise \end{cases} \tag{2}$$

- Protocol (c.f. Laneman):
  - Source TX a rate $2r$ code during the first slot.
  - If relay decodes correctly, it will TX the same codeword during second slot.
  - *SNR accumulation* at the destination.
- Outage event region:

$$\begin{aligned} \mathcal{A} &= \{C^*_{MRC} \leq r\} \\ &= \Big\{ \underbrace{[(C(\gamma_{s,r}) \geq 2r) \cap (C(\gamma_{s,d}+\gamma_{r,d}) < 2r)]}_{\text{Relay not in outage}} \cup \underbrace{[(C(\gamma_{s,r}) < 2r) \cap (C(\gamma_{s,d}) < 2r)]}_{\text{Relay in outage}} \Big\} \end{aligned} \tag{3}$$

Decode-Forward Relaying with CC

- Capacity:

$$C^*_{CC} = \begin{cases} \min\{\alpha C(\gamma_{s,d}) + \bar{\alpha} C(\gamma_{r,d}), \alpha C(\gamma_{s,r})\} & : \quad \gamma_{s,d} < \gamma_{s,r} \\ \alpha C(\gamma_{s,d}) & : \quad otherwise \end{cases} \tag{4}$$

- Protocol:
  - Source TX first part of codeword at rate r/α during first slot.
  - If relay decodes correctly, it will TX the second part of codeword at rate $r/\bar{\alpha}$ during second slot.
  - *Information accumulation* at the destination.
- Outage event region:

$$\begin{aligned} \mathcal{A} &= \{C^*_{CC} \leq r\} \\ &= \Big\{ \underbrace{\Big[\Big(C(\gamma_{s,r}) \geq \frac{r}{\alpha}\Big) \cap (\alpha C(\gamma_{s,d}) + \bar{\alpha} C(\gamma_{r,d}) < r)\Big]}_{\text{Relay not in outage}} \cup \underbrace{\Big[\Big(C(\gamma_{s,r}) < \frac{r}{\alpha}\Big) \cap \Big(C(\gamma_{s,d}) < \frac{r}{\alpha}\Big)\Big]}_{\text{Relay in outage}} \Big\} \end{aligned} \tag{5}$$

Amplify-Forward Relaying

- Capacity:

$$C^*_{AF} = \frac{1}{2}C(\gamma_{s,d} + \gamma_{s,r,d}) \tag{6}$$

  - $\gamma_{s,r,d}$ depends on the amplifier gain at the relay.
  - With an optimal gain this is:

$$\gamma_{s,r,d} = \frac{\gamma_{s,r}\gamma_{r,d}}{\gamma_{s,r} + \gamma_{r,d}} \tag{7}$$

- Protocol:
  - Source TX a rate $2r$ code during the first slot.
  - Relay amplifies its received signal by relay gain G, and forwards during the second slot.
  - SNR accumulation.
- Outage event region:

$$\mathcal{A} = \{C^*_{AF} \leq r\} \tag{8}$$

Source Adaptive Protocol A with MRC

- Capacity:

$$C^*_{SA1} = \begin{cases} \frac{1}{2}C(2\gamma_{s,d}) & \text{if } \gamma_{s,r} \leq 2^{4r} - 1 \\ \frac{1}{2}C(\gamma_{s,d} + \gamma_{r,d}) & \text{otherwise} \end{cases} \tag{9}$$

- Protocol (c.f. Laneman):
  - The source TX a rate $2r$ code during the first slot.
  - If $C(\gamma_{s,r}) \geq 2r$, the relay TX the same codeword during the second slot.
  - Otherwise, the source TX the same codeword during the second slot.
- Outage event region:

$$\begin{aligned} \mathcal{A} &= \{C^*_{SA1} \leq r\} \\ &= \Big\{ \underbrace{[(C(\gamma_{s,r}) \geq 2r) \cap (C(\gamma_{s,d} + \gamma_{r,d}) < 2r)]}_{\text{Relay not in outage}} \cup \underbrace{[(C(\gamma_{s,r}) < 2r) \cap (C(2\gamma_{s,d}) < 2r)]}_{\text{Relay in outage}} \Big\} \end{aligned} \tag{10}$$

Source Adaptive Protocol A with CC

- Capacity:

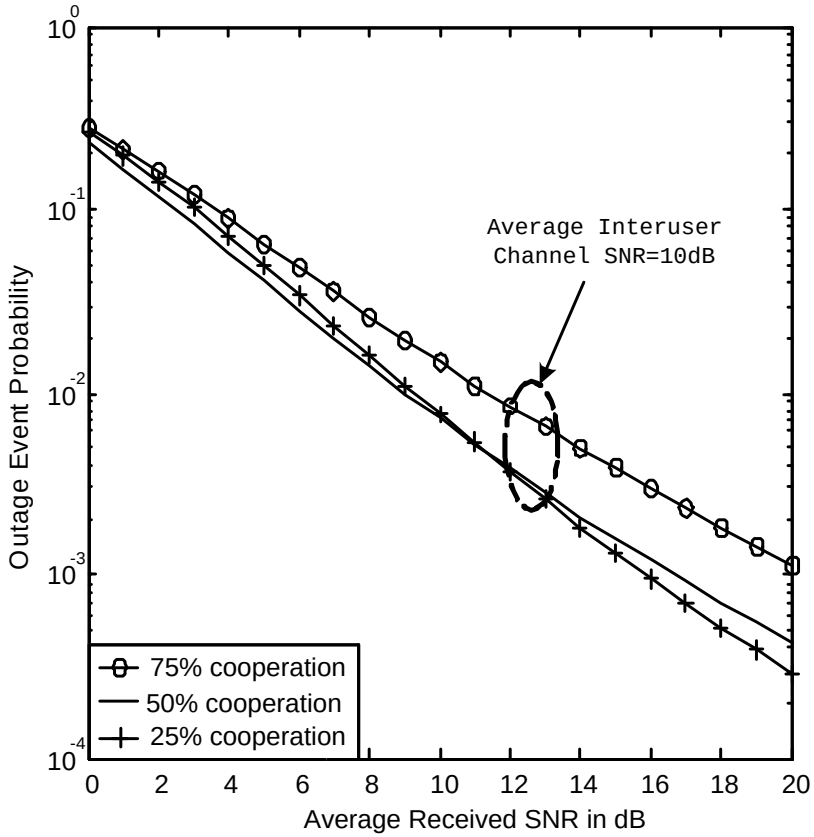
$$C^*_{SA2} = \begin{cases} C(\gamma_{s,d}) & \text{if } \gamma_{s,r} \leq 2^{2r/\alpha} - 1 \\ \alpha C(\gamma_{s,d}) + \bar{\alpha} C(\gamma_{r,d}) & \text{otherwise} \end{cases} \tag{11}$$

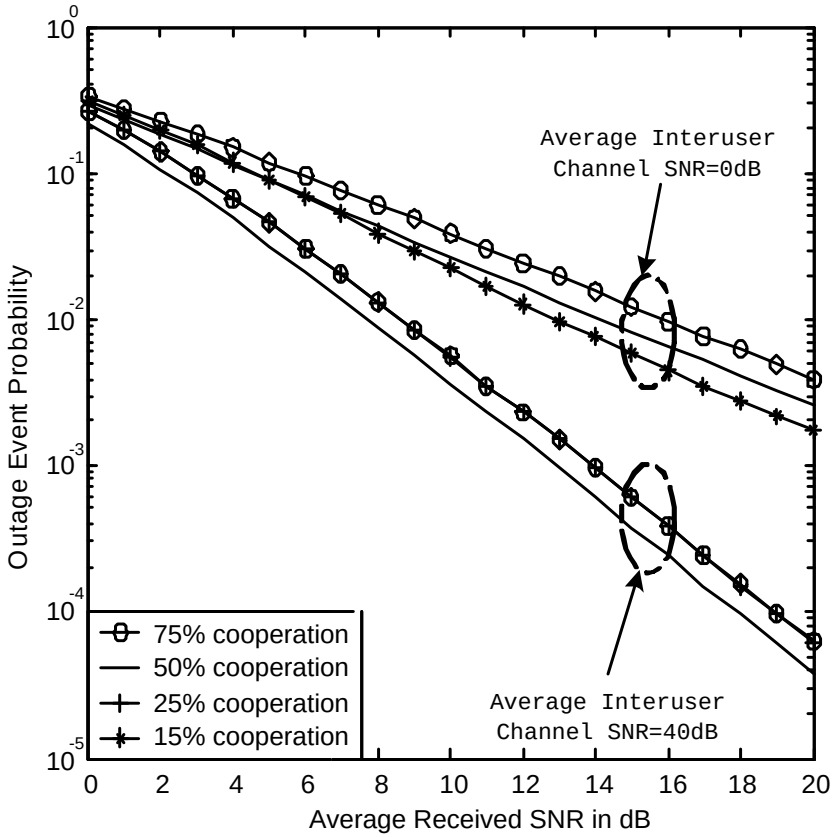
- Protocol:
  - Source TX first part of codeword at rate r/α during first slot.
  - If $C(\gamma_{s,r}) \geq 2r$, the relay will TX the second part of codeword at rate $r/\bar{\alpha}$ during second slot.
  - Otherwise, the source will transmit the remainder.
- Outage event region:

$$\begin{aligned} \mathcal{A} &= \{C^*_{SA2} \leq r\} \\ &= \Big\{ \underbrace{\Big[\Big(C(\gamma_{s,r}) \geq \frac{r}{\alpha}\Big) \cap (\alpha C(\gamma_{s,d}) + \bar{\alpha}\gamma_{r,d}) < r)\Big]}_{\text{Relay not in outage}} \cup \underbrace{\Big[\Big(C(\gamma_{s,r}) < \frac{r}{\alpha}\Big) \cap (C(\gamma_{s,d}) < r)\Big]}_{\text{Relay in outage}} \Big\} \end{aligned} \tag{12}$$

Figure 1: User Cooperative Coding (c.f Hunter & Nostratinia)

- Interchannel SNR: $\Gamma_{s,r}$
- Percent user cooperation: $\bar{\alpha} = 1 - \alpha$
- Protocol: SA-CC.





Source Adaptive Protocol B with MRC

- Capacity:

$$\begin{aligned} C^*_{SB1} &= \max\{C^*_{MRC}, C^*_{DTR}\} \\ &= \frac{1}{2}\max\left\{\min\left\{C(\gamma_{s,d}+\gamma_{r,d}), C(\gamma_{s,r})\right\}, C(2\gamma_{s,d})\right\} \end{aligned}$$

$C^*_{DTR} \triangleq \frac{1}{2}C(2\gamma_{s,d})$ is the capacity of direct transmission with MRC

- Protocol:
  - The source TX a $2r$ code during the first slot.
  - If $C(\gamma_{s,r}) > 2r$ and $\gamma_{r,d} > \gamma_{s,d}$, the relay TX the same codeword during the second block.
  - Otherwise, the source TX the same codeword during the second block
- Outage event region:

$$\begin{aligned} \mathcal{A} &= \{C^*_{SB1} \leq r\} \\ &= \{[(C(\gamma_{s,r}) < 2r) \cap (C(2\gamma_{s,d}) < 2r)] \cup [(C(\gamma_{s,r}) > 2r) \cap (C(\gamma_{s,d}+\gamma_{r,d}) < 2r) \cap (C(2\gamma_{s,d}) < 2r)]\} \end{aligned} \tag{13}$$

Source Adaptive Protocol B with CC

- Capacity:

$$\begin{aligned} C^*_{SB2} &= \max\{C^*_{CC}, C^*_{DT}\} \\ &= \max\left\{\min\left\{\alpha C(\gamma_{s,d}) + \bar{\alpha} C(\gamma_{r,d}), \alpha C(\gamma_{s,r})\right\}, C(\gamma_{s,d})\right\} \end{aligned} \tag{14}$$

- Protocol:
  - The source TX the first part of the codeword at rate r/α during the first slot.
  - If $C(\gamma_{s,r}) \geq 2r$ and $\alpha C(\gamma_{s,d}) + \bar{\alpha} C(\gamma_{r,d}) > C(\gamma_{s,d})$, the relay TX the remainder of the codeword at rate $r/\bar{\alpha}$ during the second slot.
  - Otherwise, the source TX the remainder of the codeword during the second slot.
- Outage event region:

$$\begin{aligned} \mathcal{A} &= \{C^*_{SB2} \leq r\} \\ &= \Big\{\Big[\Big(C(\gamma_{s,r}) < \frac{r}{\alpha}\Big) \cap (C(\gamma_{s,d}) < r)\Big] \\ &\quad \cup \Big[\Big(C(\gamma_{s,r}) > \frac{r}{\alpha}\Big) \cap (\alpha C(\gamma_{s,d}) + \bar{\alpha} C(\gamma_{r,d}) < r) \cap (C(\gamma_{s,d}) < r)\Big]\Big\}, \end{aligned} \tag{15}$$

Relay Adaptive Protocol

- Capacity:

$$\begin{aligned} C_{RA}^{*} &= \max\{C_{CC}^{*}, C_{AF}^{*}\} \\ &= \max\left\{\min\left\{\frac{1}{2}C(\gamma_{s,d}) + \frac{1}{2}C(\gamma_{r,d}), \frac{1}{2}C(\gamma_{s,r})\right\}, \frac{1}{2}C\left(\gamma_{s,d} + \frac{\gamma_{r,d}\gamma_{s,r}}{\gamma_{r,d}+\gamma_{s,r}}\right)\right\}. \end{aligned} \tag{16}$$

- Protocol:
  - The source TX during the first slot, while the relay TX during the second slot.
  - If $\gamma_{s,r} < 2^{4r} - 1$, the relay will use amplify-forward relaying.
  - If $\gamma_{s,r} \geq 2^{4r} - 1$, the relay will decode and forward the source message using incremental redundancy.

- Outage event region:

$$\begin{aligned} \mathcal{A} &= \{C_{RA}^{*} \leq r\} \\ &= \Bigg\{\left[(C(\gamma_{s,r}) < 2r) \cap \left(C\left(\gamma_{s,d} + \frac{\gamma_{r,d}\gamma_{s,r}}{\gamma_{r,d}+\gamma_{s,r}}\right) < 2r\right)\right] \cup \\ &\quad [(C(\gamma_{s,r}) > 2r) \cap (C(\gamma_{s,d}) + C(\gamma_{r,d}) < 2r)]\Bigg\} \end{aligned} \tag{17}$$

Comparison of Different Protocols

- Wireless propagation model
  - The received power P_r at distance d_m is related to transmitted power P_t by

$$P_r = \left(\frac{c}{4\pi d_o f_c}\right)^2 \frac{P_t}{d_m^n} = 10^{-4}\frac{P_t}{d_m^3} \tag{18}$$

    where $f_c = 2.4$ GHz, $d_o = 1$ m, and path loss coefficient $n = 3$.
  - Define "transmit" SNR as $\frac{P_t}{N}$
- independent quasi-static Rayleigh fading.
- Assume source and destination are separated by 10 m.
- Assume a single relay lies halfway between source and destination.
- Find two dimensional contours of source/relay transmit SNRs required to achieve desired OEP.

Figure 2: Transmit SNR of Source Adaptive Protocols Required to Achieve OEP = 10^{-2}

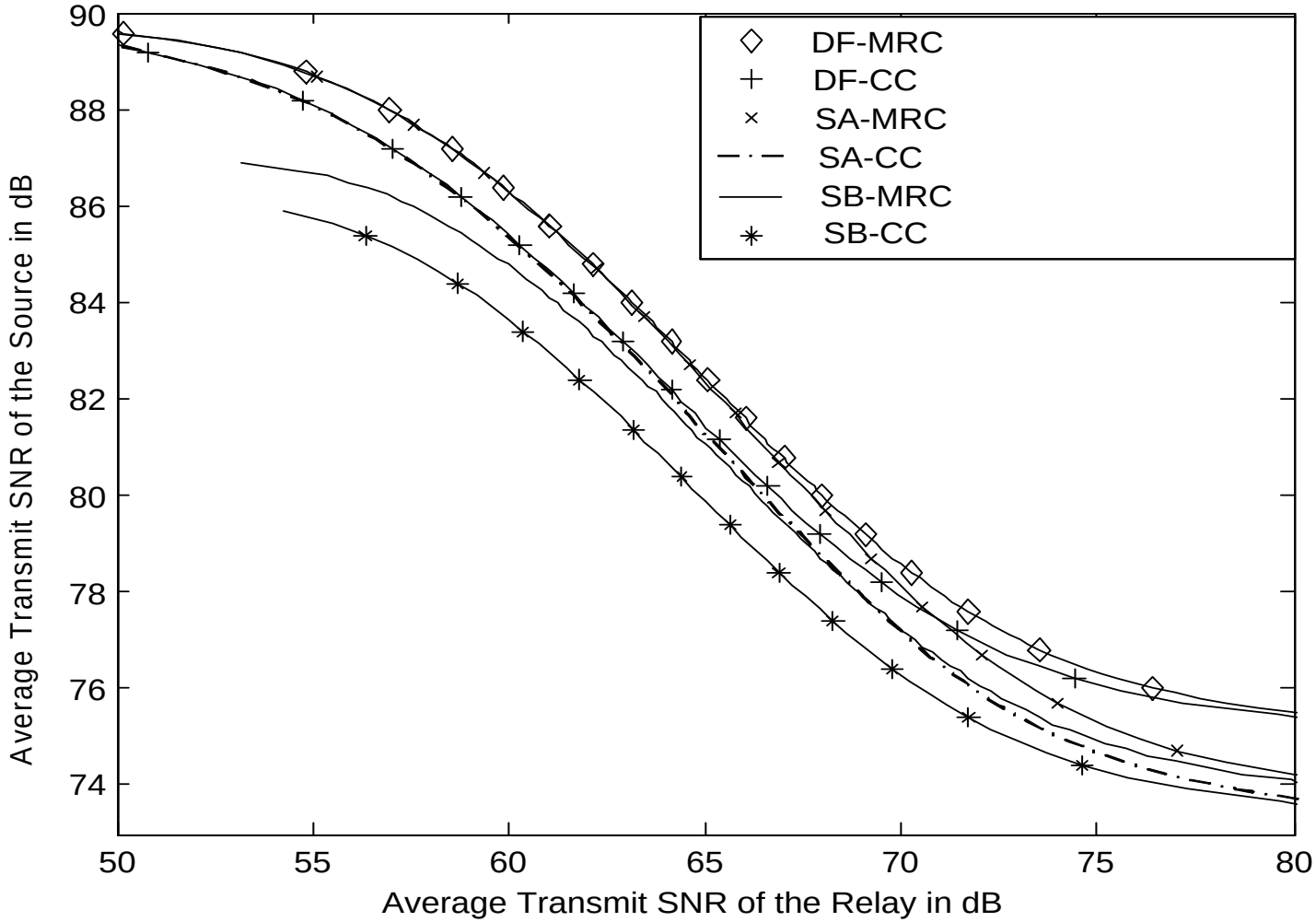
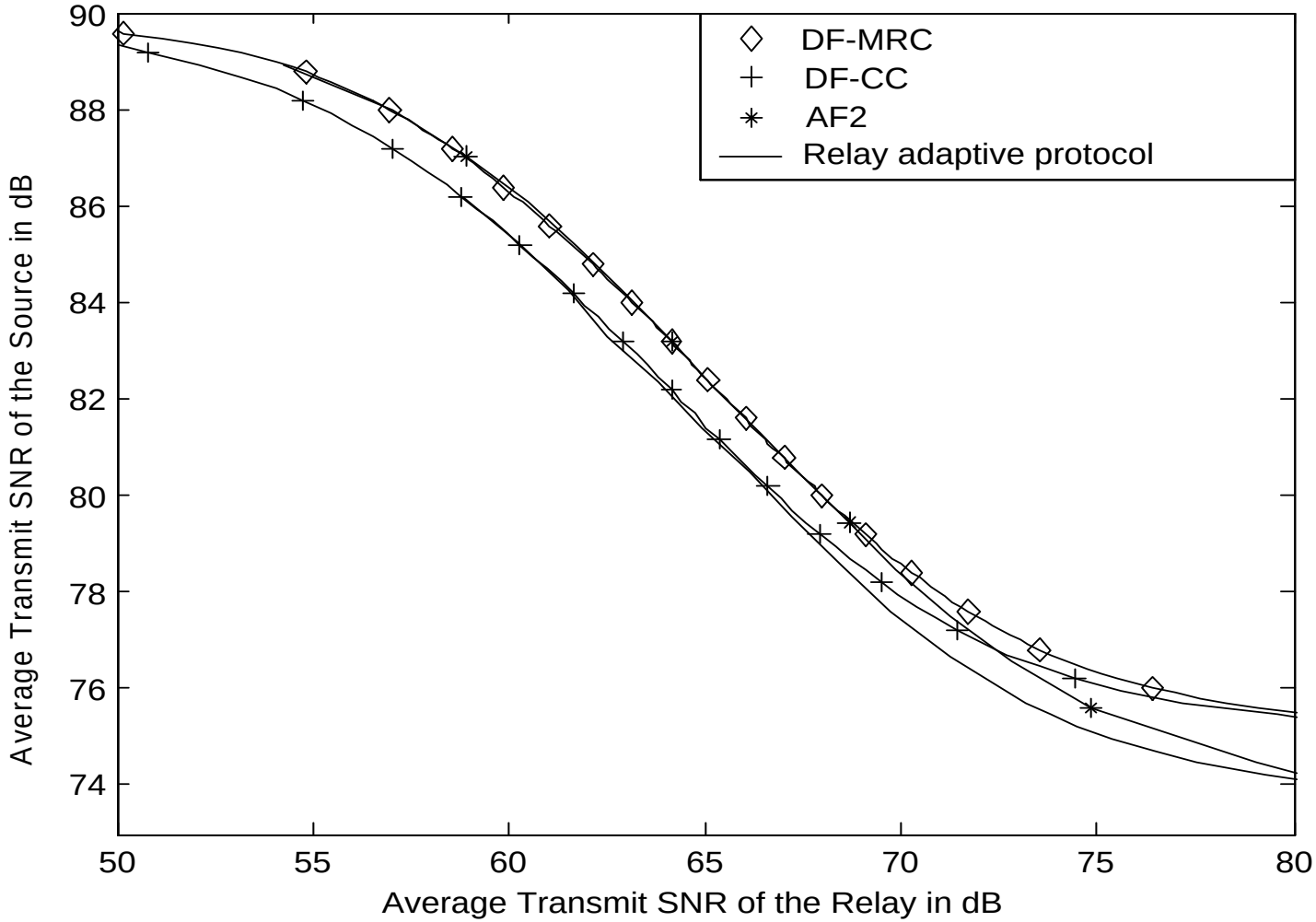


Figure 3: Transmit SNR of Relay Adaptive Protocol Required to Achieve OEP = 10^{-2}



Conclusions

- Message # 1: Protocols with incremental redundancy and code combining outperform their counterparts with repetition coding and diversity combining.
  - Difference is on the order of 1 dB.
  - With incremental redundancy, can adjust α.
- Message # 2: Adaptive protocols are more energy efficient.
  - SA uses $\gamma_{s,r}$ and is more efficient than nonadaptive strategies.
    - Only beneficial at low $\gamma_{s,r}$.
  - SB uses all three SNRs and is more efficient than SA.
- Extensions:
  - Adaptively adjust α.
  - Combination of source and relay adaptive.
  - Networks with multiple relays.
  - Allowing ARQ to guide the adaptation.