Example Problems for Chapters 1, 2 & 3

The problem is on the first slide and the solution is on the next slide.

I highly recommend that you try the problem before looking at the solution.

Each of the following equations was given by a student during an examination. Do a dimensional analysis of each equation and explain why the equation can't be correct.

(a) $\frac{1}{2}mv^2 = \frac{1}{2}mv_0^{\,2} + \sqrt{mgh}$

(b) $v = v_0 + at^2$

(c) $ma = v^2$

In the equation $\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + \sqrt{mgh}$, $[mv^2] = [mv_0^2] = \mathrm{M}\left(\frac{\mathrm{L}}{\mathrm{T}}\right)^2 = \frac{\mathrm{ML}^2}{\mathrm{T}^2}$

while $\left[\sqrt{mgh}\,\right] = \sqrt{\mathrm{M}\left(\frac{\mathrm{L}}{\mathrm{T}^2}\right)\mathrm{L}} = \frac{\mathrm{M}^{\frac{1}{2}}\mathrm{L}}{\mathrm{T}}$. Thus, the equation is $\boxed{\text{dimensionally incorrect}}$.

In $v = v_0 + at^2$, $[v] = [v_0] = \frac{\mathrm{L}}{\mathrm{T}}$ but $[at^2] = [a][t^2] = \left(\frac{\mathrm{L}}{\mathrm{T}^2}\right)\left(\mathrm{T}^2\right) = \mathrm{L}$.

Hence, this equation is $\boxed{\text{dimensionally incorrect}}$.

In the equation $ma = v^2$, we see that $[ma] = [m][a] = \mathrm{M}\left(\frac{\mathrm{L}}{\mathrm{T}^2}\right) = \frac{\mathrm{ML}}{\mathrm{T}^2}$

while $[v^2] = \left(\frac{\mathrm{L}}{\mathrm{T}}\right)^2 = \frac{\mathrm{L}^2}{\mathrm{T}^2}$

Therefore, this equation is also $\boxed{\text{dimensionally incorrect}}$.

The speed of light is now defined to be 2.99792458×10^8 m/s.

(a) Express the speed of light to three significant figures.

(b) Express the speed of light to five significant figures.

(c) Express the speed of light to seven significant figures.

$c = 2.997\ 924\ 58 \times 10^8$ m/s

(a) Rounded to 3 significant figures: $c = \boxed{3.00 \times 10^8 \text{ m/s}}$

(b) Rounded to 5 significant figures: $c = \boxed{2.997\ 9 \times 10^8 \text{ m/s}}$

(c) Rounded to 7 significant figures: $c = \boxed{2.997\ 925 \times 10^8 \text{ m/s}}$

A firkin is an old British unit of volume equal to 9 gallons. How may cubic meters are there in 6.00 firkins?

$$6.00 \text{ firkins} = 6.00 \cancel{\text{ firkins}}\left(\frac{9 \cancel{\text{ gal}}}{1 \cancel{\text{ firkin}}}\right)\left(\frac{3.786 \cancel{\text{ L}}}{1 \cancel{\text{ gal}}}\right)\left(\frac{10^3 \cancel{\text{ cm}^3}}{1 \cancel{\text{ L}}}\right)\left(\frac{1 \text{ m}^3}{10^6 \cancel{\text{ cm}^3}}\right) = \boxed{0.204 \text{ m}^3}$$

(a) **About** how many microorganisms are found in the human intestinal tract? (A typical bacterial length scale is one micron = 10^{-6} m. **Estimate** the intestinal volume and assume bacteria occupy one hundredth of it.)

(b) Discuss your answer to part (a). Are these bacteria beneficial, dangerous, or neutral? What functions could they serve?

(a) Assume that a typical intestinal tract has a length of about 7 m and average diameter of 4 cm. The estimated total intestinal volume is then

$$V_{\text{total}} = A\ell = \left(\frac{\pi d^2}{4}\right)\ell = \frac{\pi(0.04\text{ m})^2}{4}(7\text{ m}) = 0.009\text{ m}^3$$

The approximate volume occupied by a single bacteria is

$$V_{\text{bacteria}} \sim (\textit{typical length scale})^3 = \left(10^{-6}\text{ m}\right)^3 = 10^{-18}\text{ m}^3$$

If it is assumed that bacteria occupy one hundredth of the total intestinal volume, the estimate of the number of microorganisms in the human intestinal tract is

$$n = \frac{V_{\text{total}}/100}{V_{\text{bacteria}}} = \frac{\left(0.009\text{ m}^3\right)/100}{10^{-18}\text{ m}^3} = 9\times10^{13} \quad \text{or} \quad n \sim \boxed{10^{14}}$$

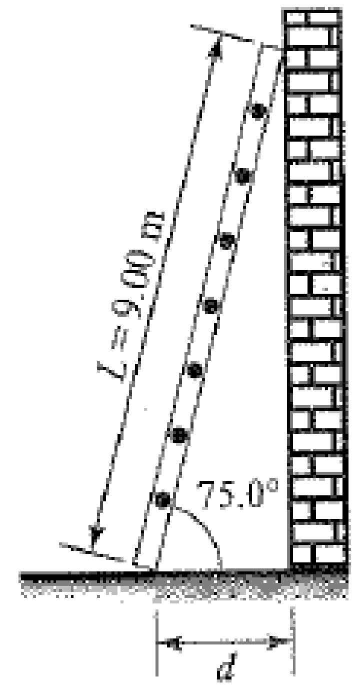
(b) The large value of the number of bacteria estimated to exist in the intestinal tract means that they are probably not dangerous. Intestinal bacteria help digest food and provide important nutrients. Humans and bacteria enjoy a mutually beneficial symbiotic relationship.

A ladder 9.00 m long leans against the side of a building. If the ladder is inclined at an angle of 75.0° to the horizontal, what is the horizontal distance from the bottom of the ladder to the building?

From the diagram, $\cos(75.0°) = d/L$

Thus,

$$d = L\cos(75.0°) = (9.00\text{ m})\cos(75.0°) = \boxed{2.33\text{ m}}$$



Light travels at a speed of about 3.0×10^8 m/s. (a) How many miles does a pulse of light travel in a time interval of 0.1 s, which is about the blink of an eye?

$\Delta x = ?$

(b) Compare this distance to the diameter of Earth. (Use 6.38×10^6 m for the radius of the Earth.)

$$\frac{\Delta x}{D_E} =$$

At constant speed, $c = 3\times10^8$ m/s, the distance light travels in 0.1 s is

$$\Delta x = c(\Delta t) = (3\times10^8\ \text{m/s})(0.1\ \text{s}) = (3\times10^7\ \cancel{\text{m}})\left(\frac{1\ \text{mi}}{1.609\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{km}}}{10^3\ \cancel{\text{m}}}\right) = \boxed{2\times10^4\ \text{mi}}$$

Comparing this to the diameter of the Earth, D_E, we find

$$\frac{\Delta x}{D_E} = \frac{\Delta x}{2R_E} = \frac{3.0\times10^7\ \cancel{\text{m}}}{2(6.38\times10^6\ \cancel{\text{m}})} \approx \boxed{2.4} \quad \text{(with } R_E = \text{Earth's radius)}$$

The average person passes out at an acceleration of 7*g* (that is, seven times the gravitational acceleration on Earth). Suppose a car is designed to accelerate at this rate. How much time would be required for the car to accelerate from rest to 60.0 miles per hour? (The car would need rocket boosters!)

From $a = \Delta v/\Delta t$, the required time is seen to be

$$\Delta t = \frac{\Delta v}{a} = \left(\frac{60.0\ \text{mi/h} - 0}{7g}\right)\left(\frac{1g}{9.80\ \text{m/s}^2}\right)\left(\frac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right) = \boxed{0.391\ \text{s}}$$

A car is traveling due east at 25.0 m/s at some instant. If its constant acceleration is 0.750 m/s^2 due east, find its velocity after 8.50 s have elapsed. B) If its constant acceleration is 0.750 m/s2 due west, find its velocity after 8.50 s have elapsed.

We choose eastward as the positive direction so the initial velocity of the car is given by $v_0 = +25.0\ \text{m/s}$.

(a) In this case, the acceleration is $a = +0.750\ \text{m/s}^2$ and the final velocity will be

$$v = v_0 + at = +25.0\ \text{m/s} + \left(+0.750\ \text{m/s}^2\right)(8.50\ \text{s}) = +31.4\ \text{m/s}$$

or

$$v = \boxed{31.4\ \text{m/s eastward}}$$

(b) When the acceleration is directed westward, $a = -0.750\ \text{m/s}^2$, the final velocity is $v = v_0 + at = +25.0\ \text{m/s} + \left(-0.750\ \text{m/s}^2\right)(8.50\ \text{s}) = +18.6\ \text{m/s}$, or $v = \boxed{18.6\ \text{m/s eastward}}$.

It is possible to shoot an arrow at a speed as high as 100 m/s.

If friction is neglected, how high would an arrow launched at this speed rise if shot straight up?
(b) How long would the arrow be in the air?

(a) For the upward flight of the arrow, $v_0 = +100\ \text{m/s}$, $a = -g = -9.80\ \text{m/s}^2$, and the final velocity is $v = 0$. Thus, $v^2 = v_0^2 + 2a(\Delta y)$ yields

$$(\Delta y)_{\text{max}} = \frac{v^2 - v_0^2}{2a} = \frac{0 - (100\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = \boxed{510\ \text{m}}$$

(b) The time for the upward flight is

$$t_{\text{up}} = \frac{(\Delta y)_{\text{max}}}{\bar{v}_{\text{up}}} = \frac{2(\Delta y)_{\text{max}}}{v_0 + v} = \frac{2(510\ \text{m})}{100\ \text{m/s} + 0} = 10.2\ \text{s}$$

For the downward flight, $\Delta y = -(\Delta y)_{\text{max}} = -510\ \text{m}$, $v_0 = 0$, and $a = -9.8\ \text{m/s}^2$. Thus,

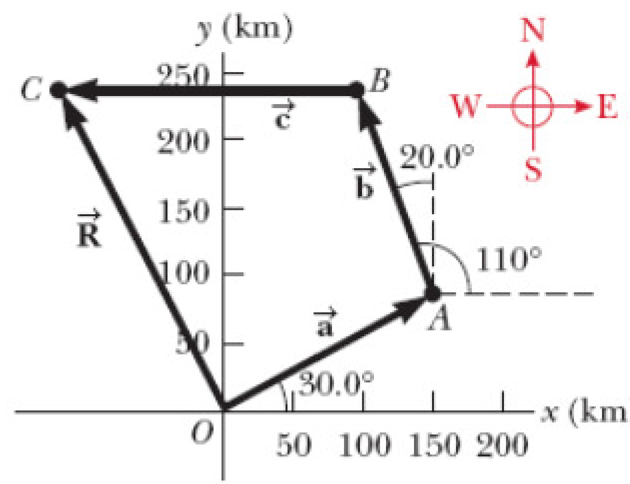
$$\Delta y = v_0 t + \tfrac{1}{2}at^2 \text{ gives } t_{\text{down}} = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-510\ \text{m})}{-9.80\ \text{m/s}^2}} = 10.2\ \text{s}$$

and the total time of the flight is $t_{\text{total}} = t_{\text{down}} + t_{\text{down}} = 10.2\ \text{s} + 10.2\ \text{s} = \boxed{20.4\ \text{s}}$.

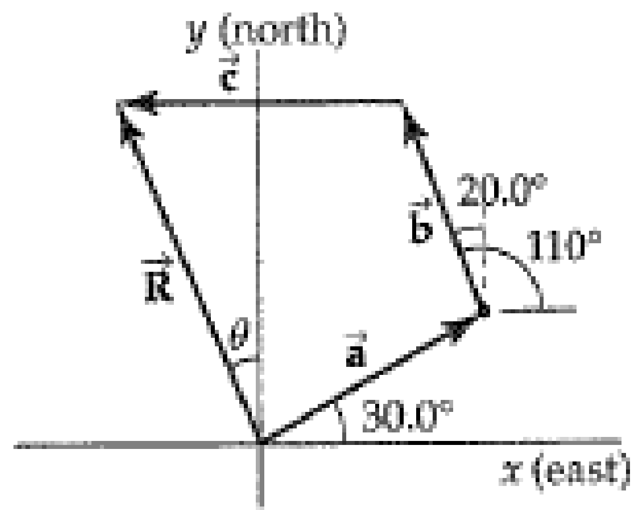
A commuter airplane starts from an airport and takes the route shown in the figure below. The plane first flies to city *A*, located 175 km away in a direction 30.0° north of east. Next, it flies for 150 km 20.0° west of north, to city *B*. Finally, the plane flies 190 km due west, to city *C*. Find the location of city *C* relative to the location of the starting point.

Distance

Angle



The components of the displacements $\vec{a}$, $\vec{b}$, and $\vec{c}$ are



$$a_x = a \cdot \cos 30.0° = +152 \text{ km}$$

$$b_x = b \cdot \cos 110° = -51.3 \text{ km}$$

$$c_x = c \cdot \cos 180° = -190 \text{ km}$$

and

$$a_y = a \cdot \sin 30.0° = +87.5 \text{ km}$$

$$b_y = b \cdot \sin 110° = +141 \text{ km}$$

$$c_y = c \cdot \sin 180° = 0$$

Thus,

$$R_x = a_x + b_x + c_x = -89.7 \text{ km, and } R_y = a_y + b_y + c_y = +228 \text{ km}$$

so

$$R = \sqrt{R_x^2 + R_y^2} = 245 \text{ km, and } \theta = \tan^{-1}\left(\frac{|R_x|}{R_y}\right) = \tan^{-1}(1.11) = 21.4°$$

City C is $\boxed{245 \text{ km at } 21.4° \text{ W of N}}$ from the starting point.