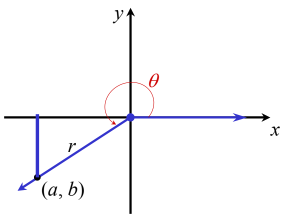
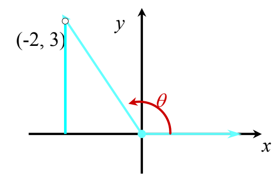


Let θ be any angle in standard position, and let (a, b) denote the coordinates of any point, except the origin (0, 0), on the terminal side of θ. If $r = \sqrt{a^2 + b^2}$ denotes the distance from (0, 0) to (a, b), then the **six trigonometric functions of** θ are defined as the ratios

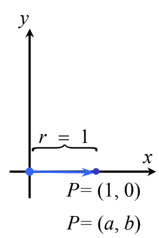
$$\sin\theta = b/r \qquad \cos\theta = a/r \qquad \tan\theta = b/a$$
$$\csc\theta = r/b \qquad \sec\theta = r/a \qquad \cot\theta = a/b$$

provided no denominator equals 0.

Find the exact value of each of the six trigonometric functions of a positive angle if (-2, 3) is a point on the terminal side.



$$a = -2, \; b = 3$$
$$r = \sqrt{a^2 + b^2} = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$$
$$\sin\theta = \frac{b}{r} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13} \qquad \csc\theta = \frac{r}{b} = \frac{\sqrt{13}}{3}$$
$$\cos\theta = \frac{a}{r} = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13} \qquad \sec\theta = \frac{r}{a} = -\frac{\sqrt{13}}{2}$$
$$\tan\theta = \frac{b}{a} = \frac{3}{-2} = -\frac{3}{2} \qquad \cot\theta = \frac{a}{b} = -\frac{2}{3}$$



$$\sin 0 = \sin 0^\circ = \frac{b}{r} = \frac{0}{1} = 0$$
$$\cos 0 = \cos 0^\circ = \frac{a}{r} = \frac{1}{1} = 1$$
$$\tan 0 = \tan 0^\circ = \frac{b}{a} = \frac{0}{1} = 0$$
$$\csc 0 = \csc 0^\circ = \frac{r}{b} = \frac{1}{0}$$
$$\sec 0 = \sec 0^\circ = \frac{r}{a} = \frac{1}{1} = 1$$
$$\cot 0 = \cot 0^\circ = \frac{a}{b} = \frac{1}{0}$$

90º ?

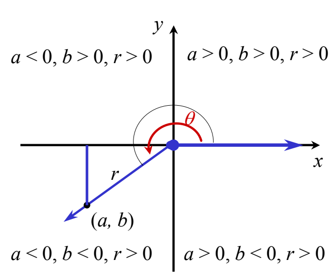
a = 0

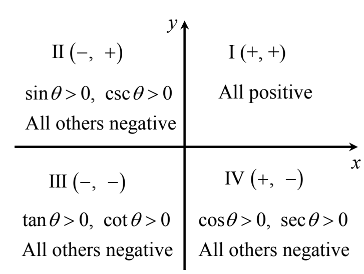
b = 1

r = 1

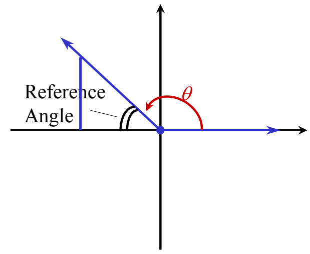
$$\sin\frac{\pi}{2} = \sin 90^\circ = \frac{b}{r} = \frac{1}{1} = 1$$
$$\cos\frac{\pi}{2} = \cos 90^\circ = \frac{a}{r} = \frac{0}{1} = 0$$
$$\tan\frac{\pi}{2} = \tan 90^\circ = \frac{b}{a} = \frac{1}{0}$$
$$\csc\frac{\pi}{2} = \csc 90^\circ = \frac{r}{b} = \frac{1}{1} = 1$$
$$\sec\frac{\pi}{2} = \sec 90^\circ = \frac{r}{a} = \frac{1}{0}$$
$$\cot\frac{\pi}{2} = \cot 90^\circ = \frac{a}{b} = \frac{0}{1} = 0$$

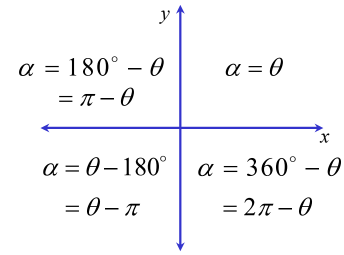
	180° (π radians)	270° ($3\pi/2$ radians)
$\sin\theta$	0	-1
$\cos\theta$	-1	0
$\tan\theta$	0	Not defined
$\csc\theta$	Not defined	-1
$\sec\theta$	-1	Not defined
$\cot\theta$	Not defined	0





Let θ denote a nonacute angle that lies in a quadrant. The acute angle formed by the terminal side of θ and either the positive x-axis or the negative x-axis is called the **reference angle** for θ.





Find the exact value of each of the following trigonometric functions using reference angles:

(a) $\cos 570^\circ$ (b) $\tan\dfrac{16\pi}{3}$

(a) $570^\circ - 360^\circ = 210^\circ = \theta$

θ in Quadrant III, so $\cos\theta < 0$

$\alpha = 210^\circ - 180^\circ = 30^\circ$

$\cos 210^\circ = -\cos 30^\circ = -\dfrac{\sqrt{3}}{2}$

(b) $\dfrac{16\pi}{3} - 2\pi = \dfrac{16\pi}{3} - \dfrac{6\pi}{3} = \dfrac{10\pi}{3}$

$\dfrac{10\pi}{3} - \dfrac{6\pi}{3} = \dfrac{4\pi}{3}$

θ is in Quadrant III, so $\tan\theta > 0$

$\alpha = \dfrac{4\pi}{3} - \pi = \dfrac{\pi}{3}$

$\tan\dfrac{16\pi}{3} = \tan\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$