



$$\begin{aligned}
 & (\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2 = \\
 & (\cos(\alpha - \beta) - 1)^2 + (\sin(\alpha - \beta) - 0)^2 \\
 & \cos^2 \alpha - 2 \cos \alpha \cos \beta + \cos^2 \beta + \\
 & \sin^2 \alpha - 2 \sin \alpha \sin \beta + \sin^2 \beta = \\
 & \cos^2(\alpha - \beta) - 2 \cos(\alpha - \beta) + 1 + \sin^2(\alpha - \beta) \\
 & 1 + 1 - 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta = \\
 & 1 + 1 - 2 \cos(\alpha - \beta)
 \end{aligned}$$

$$\boxed{\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta}$$

Sum and Difference Formulas for Cosines

$$\begin{aligned}
 \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\
 \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta
 \end{aligned}$$

Find the exact value of $\cos(105^\circ)$.

$$\begin{aligned}
 \cos(105^\circ) &= \cos(60^\circ + 45^\circ) \\
 &= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \\
 &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\
 &= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}
 \end{aligned}$$