Math 126A Name Key

Exam 1

~~Spring 2015~~ Fall 2015

Show All Work

1) Simplify each expression and eliminate any negative exponents. Assume that all letters denote positive numbers.

a) $\frac{5xy^{-2}}{10x^{-1}y^{-3}} = \frac{5x \cdot x \cdot y^3}{y^2 \cdot 10}$ (eliminated neg. exponents)

$= \frac{5x^2y^3}{10y^2}$ combined terms

$= \frac{x^2y}{2}$ cancelled

b) $(8x^{-3}y^6)^{2/3} = 8^{2/3} \cdot (x^{-3})^{2/3} \cdot (y^6)^{2/3} = (2^3)^{2/3} \cdot x^{-2} \cdot y^4$

$= 2^2 \cdot x^{-2} \cdot y^4$

$= \frac{4y^4}{x^2}$

2) Simplify the expression. Leave in radical form.

a) $\sqrt[3]{27x^8} = \sqrt[3]{3^3 \cdot x^3 \cdot x^3 \cdot x^2} = 3 \cdot x \cdot x \cdot \sqrt[3]{x^2}$

$= 3x^2 \cdot \sqrt[3]{x^2}$

b) $\sqrt{75} - \sqrt{27} = \sqrt{25 \cdot 3} - \sqrt{9 \cdot 3}$

$= 5 \cdot \sqrt{3} - 3\sqrt{3}$

$= 2\sqrt{3}$

3) Multiply and simplify $(x - 2)(x^2 + 3x + 2)$

$= x(x^2 + 3x + 2) - 2(x^2 + 3x + 2)$

$= x^3 + 3x^2 + 2x - 2x^2 - 6x - 4$

$= x^3 + x^2 - 4x - 4$

4) Factor the expressions completely

a) $2x^2 + x - 15$ $= (2x - 5)(x + 3)$

b) $x^2 - 16$ $= (x - 4)(x + 4)$

c) $x^3 - 27$ $= (x - 3)(x^2 + 3x + 9)$

Using $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

d) $2x^{1/2} + 3x^{3/2}$ $= x^{1/2}(2 + 3x)$

5) Perform the indicated operation and simplify

a) $\frac{x^2-4}{x^2-5x+6} \div \frac{x^2+3x+2}{x^2-2x-3} = \frac{x^2-4}{x^2-5x+6} \cdot \frac{x^2-2x-3}{x^2+3x+2}$

$= \frac{(x-2)(x+2)}{(x-2)(x-3)} \cdot \frac{(x-3)(x+1)}{(x+2)(x+1)} = 1$ (EVERYTHING CANCELS)

(Perform the indicated operation and simplify)

b) $\frac{1}{x^2+5x+4} - \frac{3}{x+1} = \frac{1}{(x+4)(x+1)} - \frac{3}{x+1}$

$= \frac{1}{(x+4)(x+1)} - \frac{3}{x+1} \cdot \frac{(x+4)}{(x+4)} = \frac{1-3(x+4)}{(x+1)(x+4)}$

$= \frac{1-3x-12}{(x+1)(x+4)} = \frac{-3x-11}{(x+1)(x+4)}$

6) Solve the following equations

a) $\frac{2x-2}{x+3} = \frac{2}{3}$

CROSS MULTIPLY

$3(2x-2) = 2(x+3)$

$6x-6 = 2x+6$

$4x = 12$

$x = 3$

b) $\frac{4}{x-2} + \frac{2}{x+2} = \frac{22}{x^2-4}$

$(x-2)(x+2)\left[\frac{4}{x-2} + \frac{2}{x+2}\right] = \frac{22}{x^2-4}(x-2)(x+2)$

$(x-2)(x+2)\cdot\frac{4}{x-2} + (x-2)(x+2)\cdot\frac{2}{x+2} = 22$

$4(x+2) + 2(x-2) = 22$

$4x+8 + 2x-4 = 22$

$6x+4 = 22$

$6x = 18$

$x = 3$

7) Phyllis invested 20,000, a portion earning a simple interest rate of 4% per year and the rest earning a simple interest rate of 5% per year. After 1 year the total interest earned on these investments was $875. How much money did she invest at each rate?

x dollars invested at 4%

y dollars invested at 5%

$x + y = 20{,}000 \longrightarrow y = 20{,}000 - x$

$.04x + .05y = 875$

$.04x + .05(20{,}000 - x) = 875$

$.04x + 1000 - .05x = 875$

$-.01x = -125$

$x = 12{,}500$ — invested at 4%

$y = 7500$ — invested at 5%

8) Find all solutions of the equation.

a) $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ OR $x = -2$

b) $x^2 - 2x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2-(4)(1)(-6)}}{2 \cdot 1} = \frac{2 \pm \sqrt{4+24}}{2}$$

$$= \frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm \sqrt{4 \cdot 7}}{2} = \frac{2 \pm 2\sqrt{7}}{2} = \frac{2(1 \pm \sqrt{7})}{2} = 1 \pm \sqrt{7}$$

c) $x^2 - 2x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2-4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{2 \pm \sqrt{4-20}}{2}$$

$$= \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = \frac{2(1 \pm 2i)}{2} = 1 \pm 2i$$