Math 126
Exam 2
Spring 2015

Name Key

Show ALL work

1) Find all solutions (real or complex) of $x^2 - 2x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-2) \pm \sqrt{4-20}}{2}$$

$$= \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = \frac{2(1 \pm 2i)}{2} = 1 \pm 2i$$

2) Write $\frac{3-4i}{2+i}$ in the form a + bi

$$\frac{3-4i}{2+i} \cdot \frac{2-i}{2-i} = \frac{6 - 3i - 8i + 4i^2}{4 - i^2} = \frac{6 - 11i - 4}{5}$$

$$= \frac{2 - 11i}{5} = \frac{2}{5} - \frac{11}{5}i$$

3) Find all real solutions of the equation.

a) $x^4 - 8x^2 - 9 = 0$

$u = x^2$

$u^2 - 8u - 9 = 0$

$(u-9)(u+1) = 0$

$u = 9$ or $u = -1$

$x^2 = 9$ or $x^2 = -1$ ✗

$x = \pm 3$

b) $\sqrt{2x+3} = 2x+1$

Square

$2x+3 = (2x+1)^2$

$2x+3 = 4x^2+4x+1$

$0 = 4x^2+2x-2$

$0 = 2(2x^2+x-1)$

$0 = 2(2x-1)(x+1)$

$x = \frac{1}{2}$ OR $x = -1$

check

$x = \frac{1}{2}$

$\sqrt{4} = 2$ ✓

$x = -1$

$\sqrt{1} = -1$ ✗

ANSWER $\boxed{x = \frac{1}{2}}$

c) $|3x-2| = 4$

$3x-2 = 4$ OR $3x-2 = -4$

$3x = 6$ $\quad 3x = -2$

$x = 2$ OR $x = \frac{-2}{3}$

4) Solve the inequality. Graph the solution set on the real number line.

a) $x^2 + 4x - 12 > 0$

$(x+6)(x-2) > 0$

$x = -7$ $\quad - \cdot - = +$

$x = 0$ $\quad + \cdot - = -$

$x = 3$ $\quad + \cdot + = +$

$x < -6$ OR $x > 2$

$(-\infty, -6) \cup (2, \infty)$

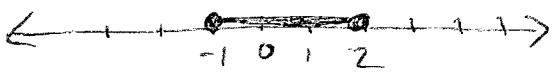
b) $|2x-1| \leq 3$

$-3 \leq 2x-1 \leq 3$

$-2 \leq 2x \leq 4$

$-1 \leq x \leq 2$

$[-1, 2]$



5) Find the intercepts and graph by plotting points $y = x^2 - 4$.

x	y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5

root if $x = 0$

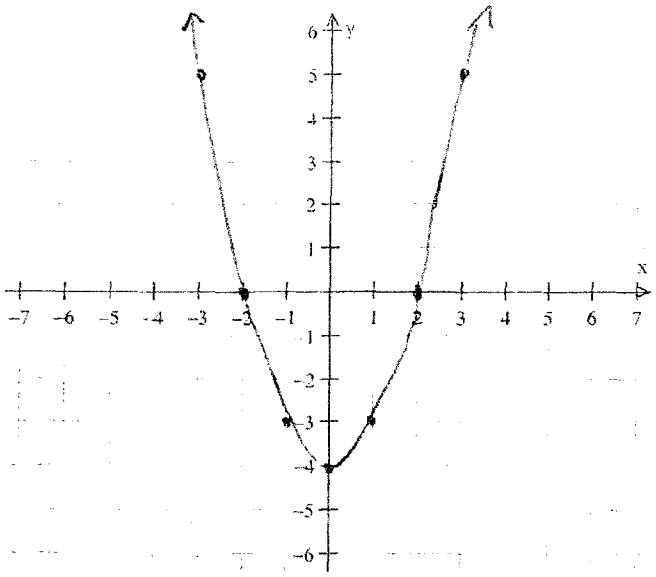
$y = -4$

if $y = 0$

$0 = x^2 - 4$

$4 = x^2$

$\pm 2 = x$



6) Let $P_1 = (2, -3)$ and $P_2 = (4, 5)$

a) Find the distance between P_1 and P_2

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4-2)^2 + (5-(-3))^2} = \sqrt{2^2 + 8^2} = \sqrt{68}$$

b) Find the midpoint of the line segment containing P_1 and P_2.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2+4}{2}, \frac{-3+5}{2}\right) = (3, 1)$$

c) What is the slope of the line containing P_1 and P_2?

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{4 - 2} = \frac{8}{2} = 4$$

d) What is the equation of the line through P_1 and P_2?

$y - y_1 = m(x - x_1)$

$y - (-3) = 4(x - 2)$

$y + 3 = 4x - 8$

$y = 4x - 11$

e) What is the equation of a line through P_1 with undefined slope?

$x = 2$

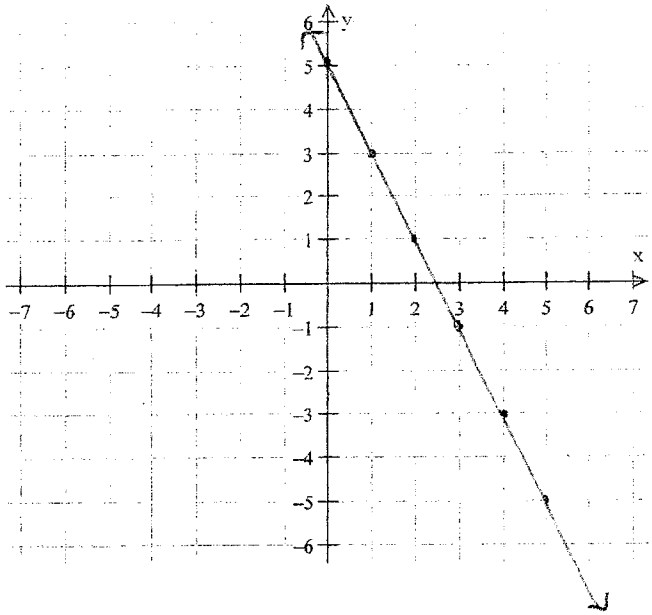
7) What is the slope of any line perpendicular to $2x + y = 6$? $\frac{1}{2}$

$y = 6 - 2x$

slope -2

need negative reciprocal

8) Graph the line containing the point (1, 3) and having slope -2.



9) What is the center and radius of the circle $(x-2)^2 + (y+1)^2 = 5$?

center $(2, -1)$

radius $\sqrt{5}$

10) Find the standard equation of the circle with general equation

$x^2 + y^2 - 8x + 2y + 8 = 0$

$\left(\frac{1}{2} \cdot 8\right)^2$

$x^2 - 8x + y^2 + 2y = -8$

$x^2 - 8x + 16 + y^2 + 2y + 1 = -8 + 16 + 1$

$(x-4)^2 + (y+1)^2 = 9$