Math 126
Exam 3
Fall 2015

Name Key

Show All Work

1) If $f(x) = 3 - x - x^2$ what is $f(x+2)$? Simplify.

$f(x+2) = 3 - (x+2) - (x+2)^2 = 3 - x - 2 - (x^2 + 4x + 4)$

$= 3 - x - 2 - x^2 - 4x - 4 = -x^2 - 5x - 3$

2) If $f(x) = \frac{x^2-1}{x^2-4}$

a) What is the domain of f(x)

$\{x \mid x \neq \pm 2\}$ $= (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

b) What are its x and y intercepts?

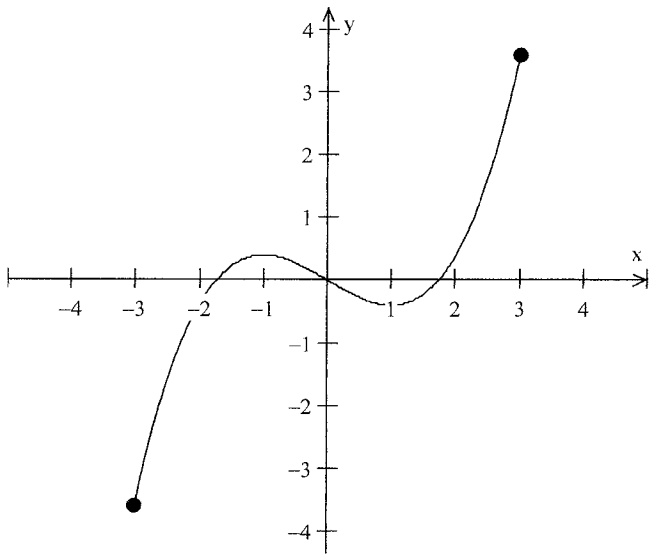
x=0 $y = \frac{1}{4}$ $(0, \frac{1}{4})$

y=0 $0 = \frac{x^2-1}{x^2-4} \rightarrow x^2 - 1 = 0 \rightarrow x = \pm 1$ $(\pm 1, 0)$

c) If f(x) even, odd, both or neither?

$f(-x) = \frac{(-x)^2-1}{(-x)^2-4} = \frac{x^2-1}{x^2-4} = f(x)$ EVEN

3) Given the graph of $g(x)$ on the right.



a) What kind of symmetry (if any) does it have?

none

b) On what interval(s) is it decreasing?

$(-1, 1)$

c) List the x coordinate(s) of the local maximum(s).

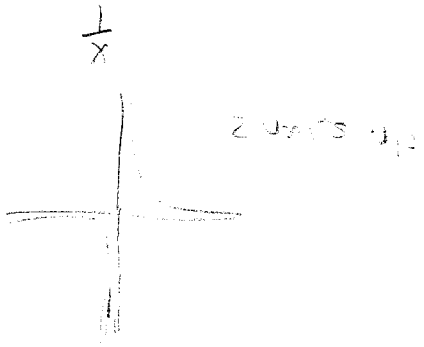
$x = -1$

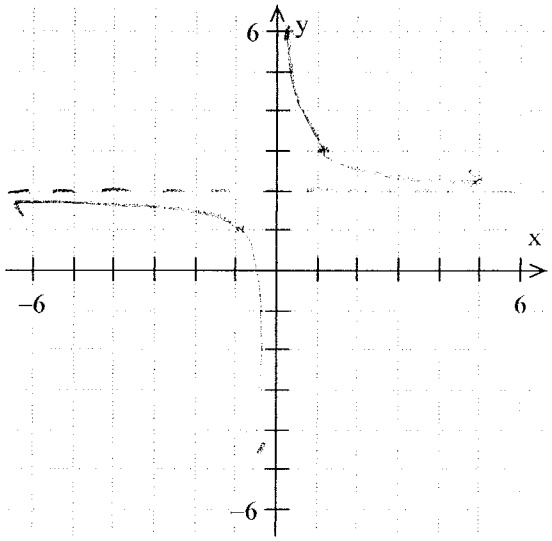
d) What is the domain of g(x).

$[-3, 3]$

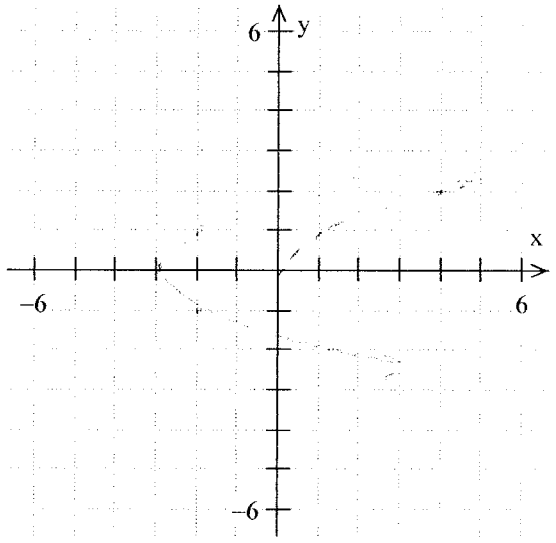
4) Graph the following functions using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages.

a) $f(x) = \frac{1}{x} + 2$

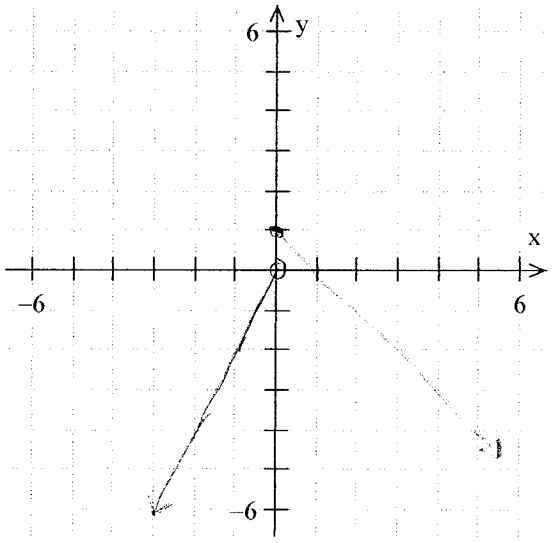




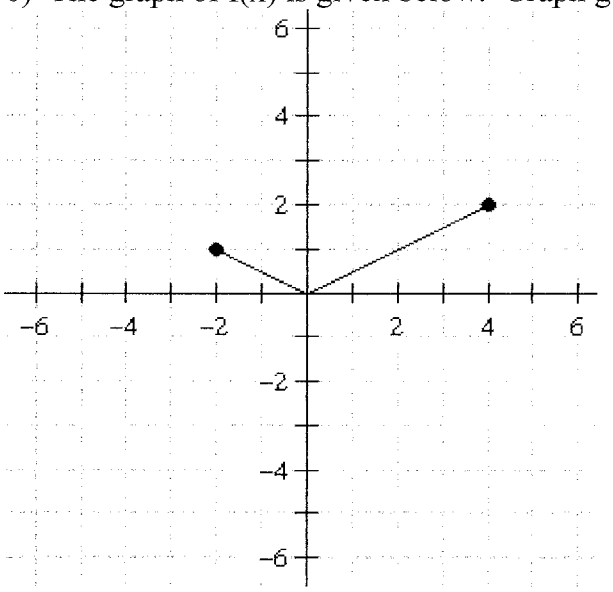
b) $g(x) = -\sqrt{x+3}$

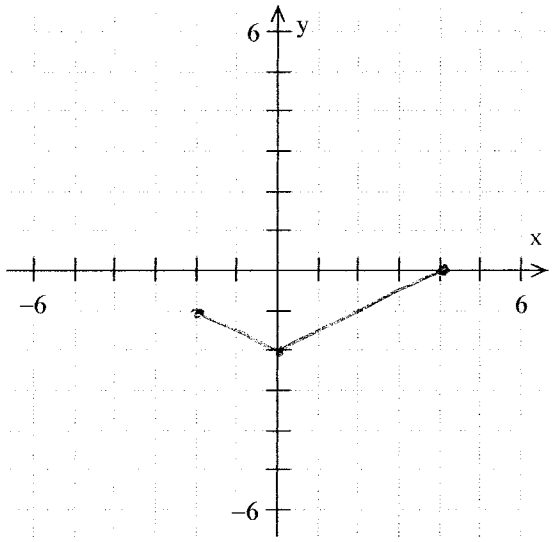


5) Graph $f(x) = \begin{cases} 2x & if\ x < 0 \\ -x + 1 & if\ x \geq 0 \end{cases}$



6) The graph of f(x) is given below. Graph g(x) = f(x) - 2





7) If $f(x) = \frac{2}{x}$ and $g(x) = \frac{1}{x-3}$

a) Find f ∘ g and its domain

$f(g(x)) = f\left(\frac{1}{x-3}\right) = \frac{2}{\frac{1}{x-3}} = 2(x-3)$

Domain $x \neq 3$

b) Find g ∘ f and its domain.

$g(f(x)) = g\left(\frac{2}{x}\right) = \frac{1}{\frac{2}{x}-3} = \frac{1}{\frac{2-3x}{x}} = \frac{x}{2-3x}$

$x \neq 0 \quad 2-3x \neq 0$

$2 \neq 3x$

Domain $\left\{x \mid x \neq 0, x \neq \frac{2}{3}\right\}$

c) Find $g^{-1}(x)$

$y = \frac{1}{x-3}$

Switch

$x = \frac{1}{y-3}$

$x(y-3) = 1$

$xy - 3x = 1$

$xy = 1+3x$

$y = \frac{1+3x}{x}$

$g^{-1}(x) = \frac{1+3x}{x}$

8) Put the quadratic function below in standard form and graph. Find the vertex and intercepts, if any.

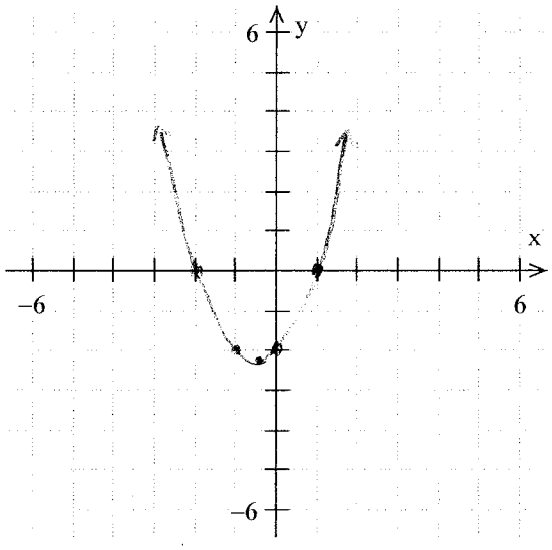
$f(x) = x^2 + x - 2$

$= (x^2 + x \quad) - 2$

$= \left(x^2 + x + \frac{1}{4}\right) - 2 - \frac{1}{4}$

$= \left(x + \frac{1}{2}\right)^2 - \frac{9}{4}$

Vertex $\left(-\frac{1}{2}, -\frac{9}{4}\right)$



Intercepts

If $x=0$ $y=-2$ $(0,-2)$

If $y=0$ $0 = x^2 + x - 2$

$0 = (x+2)(x-1)$

$x+2=0$ or $x-1=0$

$x=-2$ or $x=1$

$(-2,0)$ or $(1,0)$

9) The weekly profit from selling x Whatchamacallits is given by $P(x) = 800x - 20x^2$

a) How many Whatchamacallits must the company sell to maximize profit?

$x = \frac{-b}{2a} = \frac{-800}{2(-20)} = \frac{-800}{-40} = 20$

b) What is the maximum profit?

$P(20) = 800(20) - 20(20)^2$

$= 16{,}000 - 8000$

$= 8000$