

5) Let $\mathbf{a} = \langle 8, 4, -1 \rangle$ and $\mathbf{b} = \langle 7, -6, 6 \rangle$

a) Find $|\mathbf{a}|$ and $|\mathbf{b}|$

$$|\vec{a}| = \sqrt{64+16+1} = \sqrt{81} = 9$$

$$|\vec{b}| = \sqrt{49+36+36} = \sqrt{121} = 11$$

b) Find $\mathbf{a} \cdot \mathbf{b}$

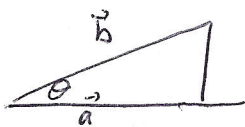
$$\vec{a} \cdot \vec{b} = 56 - 24 - 6 = 26$$

c) Find the angle between \mathbf{a} and \mathbf{b}

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{26}{9 \cdot 11} = \frac{26}{9}$$

$$\theta = 74.8^\circ$$

d) Find the scalar projection of \mathbf{b} onto \mathbf{a} .



$$\text{comp}_{\vec{a}} \vec{b} = |\vec{b}| \cos \theta$$

$$= 11 \cdot \frac{26}{99} = \frac{26}{9} \approx 2.89$$

e) Find the vector projection of \mathbf{b} onto \mathbf{a} .

$$\left. \begin{array}{l} (\text{comp}_{\vec{a}} \vec{b}) \cdot \frac{\vec{a}}{|\vec{a}|} \\ \text{or} \\ \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|} \cdot \frac{\vec{a}}{|\vec{a}|} \end{array} \right\} = \frac{26}{9} \cdot \frac{\langle 8, 4, -1 \rangle}{9} = \left\langle \frac{208}{81}, \frac{104}{81}, -\frac{26}{81} \right\rangle$$

6) a) Find a nonzero vector orthogonal to the plane through the points $(-1, 3, 1)$, $(0, 5, 2)$, $(4, 3, -1)$

$$\left. \begin{array}{l} \vec{a} = \langle 1, 2, 1 \rangle \\ \vec{b} = \langle 4, -2, -3 \rangle \end{array} \right\} \text{two vectors in the plane}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 4 & -2 & -3 \end{vmatrix} = \boxed{\langle -4, 7, -10 \rangle}$$

b) Find the area of the triangle with the above 3 points as vertices.

$$A = \frac{1}{2} |\vec{a} \times \vec{b}| = \frac{1}{2} \sqrt{16+49+100} = \frac{1}{2} \sqrt{165} = 6.42$$