Math 251 Name Key

Exam 2

Spring 2013 Show All Work

1) If $\mathbf{r}(t) = \langle t^2, \frac{t^2}{2}, t \rangle$, find

a) **T**(t)

$\vec{r}'(t) = \langle 2t, t, 1 \rangle$

$|\vec{r}'(t)| = \sqrt{4t^2 + t^2 + 1} = \sqrt{5t^2+1}$

$\vec{T}(t) = \left\langle \frac{2t}{\sqrt{5t^2+1}}, \frac{t}{\sqrt{5t^2+1}}, \frac{1}{\sqrt{5t^2+1}} \right\rangle$

b) the curvature κ

$K = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3}$

$\vec{r}' \times \vec{r}'' = \begin{vmatrix} i & j & k \\ 2t & t & 1 \\ 2 & 1 & 0 \end{vmatrix} = \langle -1, 2, 0 \rangle$

$|\vec{r}' \times \vec{r}''| = \sqrt{1+4} = \sqrt{5}$

$K = \frac{\sqrt{5}}{(5t^2+1)^{3/2}}$

2) Find the length of the curve $\mathbf{r}(t) = \langle \cos 6t, \sin 6t, 8t \rangle$ for $0 \le t \le 2$

$\vec{r}'(t) = \langle -6\sin 6t, 6\cos 6t, 8 \rangle$

$|\vec{r}'(t)| = \sqrt{36\sin^2 6t + 36\cos^2 6t + 64}$

$= \sqrt{36+64} = \sqrt{100} = 10$

$L = \int_0^2 |\vec{r}'(t)|\, dt = \int_0^2 10\, dt = 10t \Big|_0^2 = 20$