

7) Show that the following limit does not exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{7x^2 + 3y^2}$$

x-axis ( $y=0$ )

$$\lim_{(x,0) \rightarrow (0,0)} \frac{x^2}{7x^2} = \lim_{x \rightarrow 0} \frac{1}{7} = \frac{1}{7}$$

y-axis ( $x=0$ )

$$\lim_{(0,y) \rightarrow (0,0)} \frac{-y^2}{3y^2} = \lim_{y \rightarrow 0} \frac{-1}{3} = -\frac{1}{3}$$

Since the above limits do not agree the general limit does not exist

8) Find all first and second partial derivatives of  $f(x,y) = x \sin y + \frac{x}{y} - e^x$

$$f_x = \sin y + \frac{1}{y} - e^x$$

$$f_y = x \cos y - \frac{x}{y^2}$$

$$f_{xx} = -e^{-x}$$

$$f_{yy} = -x \sin y + \frac{2x}{y^3}$$

$$f_{xy} = \cos y - \frac{1}{y^2}$$

$$f_{yx} = \cos y - \frac{1}{y^2}$$

9) If  $x^2 y^3 = \sin(xy)$ , find  $dy/dx$  without using implicit differentiation

$$\frac{dy}{dx} = \frac{-f_x}{f_y} = - \left( \frac{2xy^3 - y \cos(xy)}{3x^2y^2 - x \cos(xy)} \right)$$