

- 10) Find the equation of the tangent plane to the surface $z = \sqrt{18 - 2x^2 - y^2}$ at the point $(2, 3, 1)$.

$$f_x = \frac{1}{2} (18 - 2x^2 - y^2)^{-\frac{1}{2}} \cdot (-4x) = \frac{-2x}{\sqrt{18 - 2x^2 - y^2}} \quad f_x(2, 3, 1) = \frac{-4}{1} = -4$$

$$f_y = \frac{1}{2} (18 - 2x^2 - y^2)^{-\frac{1}{2}} \cdot (-2y) = \frac{-y}{\sqrt{18 - 2x^2 - y^2}} \quad f_y(2, 3, 1) = \frac{-3}{1} = -3$$

Plane

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$z - 1 = -4(x - 2) - 3(y - 3)$$

$$z - 1 = -4x + 8 - 3y + 9$$

$$\boxed{4x + 3y + z = 18}$$

- 11) Use differentials and the formula $V = \pi r^2 h$ to estimate the amount of tin in a cylindrical tin can with diameter 5 cm and height 8 cm if the sides are 0.04 cm thick and the top and bottom are 0.06 cm thick.

$$dV \cong d\mathcal{V} = \frac{\partial V}{\partial r} dr + \frac{\partial V}{\partial h} dh$$

$$\frac{dr}{dh} \cong \frac{\Delta r}{\Delta h} = \frac{.04}{2(.06)} = .12$$

$$dV = 2\pi rh(.04) + \pi r^2(.12)$$

$$dV \Big|_{r=2.5, h=8} = 2\pi(2.5)(8)(.04) + \pi(2.5)^2(.12)$$

$$\begin{matrix} r=2.5 \\ h=8 \end{matrix}$$

$$= 1.6\pi + .75\pi = 2.35\pi \cong 7.38 \text{ cm}^3$$