12) The length l, width w and height h of a box change with time. Find the rate at which the volume is change when $l = 4$, $w = 6$ and $h = 8$, l and w are increasing at a rate of 10 m/s and h is decreasing at a rate of 1 m/s.

$$V = l \cdot w \cdot h$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial l} \cdot \frac{dl}{dt} + \frac{\partial V}{\partial w} \cdot \frac{dw}{dt} + \frac{\partial V}{\partial h} \cdot \frac{dh}{dt}$$

$$= wh\,(10) + lh\,(10) + lw\,(-1)$$

$$\left.\frac{dV}{dt}\right|_{\substack{l=4 \\ w=6 \\ h=8}} = 6 \cdot 8 \cdot 10 + 4 \cdot 8 \cdot 10 - 4 \cdot 6 = 480 + 320 - 24$$

$$= 776 \ m^3/s$$

13) Let $f(x, y, z) = 3x^2 + 4y^3 + z$ and let P = (3, 2, 1)

a) Find the gradient of f at P.

$$\nabla f = \langle f_x, f_y, f_z \rangle = \langle 6x, 12y^2, 1 \rangle$$

$$\nabla f(3,2,1) = \langle 18, 48, 1 \rangle$$

b) Find the maximum rate of change of f at the given point and the direction in which it occurs.

$$\text{max rate} = |\nabla f| = \sqrt{18^2 + 48^2 + 1^2} = \sqrt{2629} \doteq 51.3$$

in the direction $\langle 18, 48, 1 \rangle$

c) Find the directional derivative of f(x, y, z) in the direction of v = <1, 2, -2> at P.

$$|v| = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$u = \left\langle \frac{1}{3}, \frac{2}{3}, \frac{-2}{3} \right\rangle$$

$$D_u f = \nabla f \cdot u = \langle 18, 48, 1 \rangle \cdot \left\langle \frac{1}{3}, \frac{2}{3}, \frac{-2}{3} \right\rangle = 6 + 32 - \frac{2}{3} = 38 - \frac{2}{3}$$

$$= \boxed{37\frac{1}{3} = \frac{112}{3}}$$