Math 251
Exam 3 Part 1 (15 points)
Spring 2013

Name Key

Show All Work

1) Use Lagrange multipliers to find maximum and minimum values of $f(x,y,z) = x^4 + y^4 + z^4$ subject to the constraint $x^2 + y^2 + z^2 = 18$.

$\nabla f = \langle 4x^3, 4y^3, 4z^3 \rangle$ $\quad$ $\nabla g = \langle 2x, 2y, 2z \rangle$

$\nabla f = \lambda \cdot \nabla g$

$$\boxed{\begin{aligned} 4x^3 &= 2x\lambda \\ 4y^3 &= 2y\lambda \\ 4z^3 &= 2z\lambda \\ x^2+y^2+z^2 &= 18 \end{aligned}} \longrightarrow \begin{aligned} &x=0 \text{ or } 2x^2=\lambda \\ &y=0 \text{ or } 2y^2=\lambda \\ &z=0 \text{ or } 2z^2=\lambda \\ &x^2+y^2+z^2=18 \end{aligned}$$

case 1 $\quad x=y=z=0 \rightarrow$ impossible $x^2+y^2+z^2=18$

case 2 $\quad$ two variables $=0$ say x and y

$z^2 = 18$

$z = \pm\sqrt{18} = \pm 3\sqrt{2}$

points $(0,0,\pm 3\sqrt{2})$ $\quad$ by symmetry $(0, \pm 3\sqrt{2}, 0)$, $(\pm 3\sqrt{2}, 0, 0)$

$f(\text{These}) = 18^2 = 324$

case 3 $\quad$ one variable zero say x

then $2y^2 = \lambda = 2z^2$ so $y^2 = z^2$

so $x^2+y^2+z^2=18$ becomes

$2y^2 = 18$

$y^2 = 9$

$y = \pm 3$

$z = \pm 3$

points $(0, \pm 3, \pm 3)$

by symmetry $(\pm 3, 0, \pm 3)$

$(\pm 3, \pm 3, 0)$

$f(\text{these}) = 81 + 81 = 162$

case 4 $\quad$ no varable $= 0$

$2x^2 = 2y^2 = 2z^2 \rightarrow x^2 = y^2 = z^2$

so $3x^2 = 18$

$x^2 = 6$

$x = \pm\sqrt{6}$

pts $(\pm\sqrt{6}, \pm\sqrt{6}, \pm\sqrt{6})$

$f(\text{These points}) = 6^2 + 6^2 + 6^2 = 108$

MAX $= 324$

MIN $= 108$