

Show All Work

1) Find the local maximum and minimum values and saddle points of the function

$$f(x,y) = 2y^3 + 3x^2y - 9y^2 - 3x^2 + 2$$

$$f_x(x,y) = 6xy - 6x \longrightarrow f_x = 0 \rightarrow 6xy - 6x = 0 \rightarrow 6x(y-1) = 0$$

$$f_y(x,y) = 6y^2 + 3x^2 - 18y \longrightarrow f_y = 0$$

$x=0$ or $y=1$

If $x=0$

$$6y^2 + 3x^2 - 18y = 0$$

Becomes

$$6y^2 - 18y = 0$$

$$6y(y-3) = 0$$

$y=0$ or $y=3$

$(0,0)$ or $(0,3)$

If $y=1$

$$6y^2 + 3x^2 - 18y = 0$$

Becomes

$$6 + 3x^2 - 18 = 0$$

$$3x^2 - 12 = 0$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

$(2,1)$, or $(-2,1)$

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

$$= (6y-6)(12y-18) - (6x)^2$$

$D(0,0) = (-6)(-18) > 0$	$f_{xx} < 0$ c. down	LOCAL MAX AT $(0,0,2)$
$D(0,3) = (12)(18) > 0$	$f_{xx} > 0$ c. up	LOCAL MIN AT $(0,3,-25)$
$D(2,1) = 0 - 144 < 0$		SADDLE POINT $(2,1,-5)$
$D(-2,1) = 0 - 144 < 0$		SADDLE POINT $(-2,1,-5)$

Scratch

$$f(0,3) = 54 - 81 + 2 = -25$$

$$f(2,1) = 2 + 12 - 9 - 12 + 2 = -5$$

$$f(-2,1) = 2 + 12 - 9 - 12 + 2 = -5$$