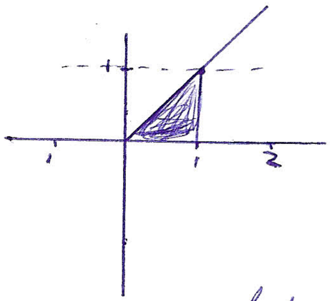
4) Sketch the region of integration, change the order of integration, and integrate.

$$\int_0^1 \int_y^1 e^{-x^2}\,dx\,dy$$

bounds $y=0$ $\quad x=y$
$\quad\quad\quad y=1$ $\quad x=1$



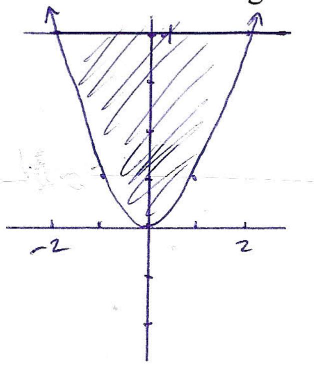
$$\int_0^1 \int_0^x e^{-x^2}\,dy\,dx = \int_0^1 e^{-x^2}\cdot y\Big|_{y=0}^{y=x} dx = \int_0^1 x e^{-x^2}\,dx$$

let $u=-x^2$
$du=-2x\,dx$
$-\frac{1}{2}du = x\,dx$

$$= \int_0^{-1} -\frac{1}{2} e^u\,du = -\frac{1}{2}e^u\Big|_0^{-1}$$

$$= -\frac{1}{2}e^{-1} + \frac{1}{2}$$

5) Use a double integral to find the volume of the region under the surface of $z = 1 + x^2y^3$ and above the region enclosed by $y = x^2$ and $y = 4$.



$$\int_{-2}^{2}\int_{x^2}^{4} (1 + x^2y^3)\,dy\,dx$$

$$= \int_{-2}^{2} \left(y + \frac{x^2 \cdot y^4}{4}\right)\Big|_{y=x^2}^{y=4} dx$$

$$= \int_{-2}^{2} \left[(4 + 64x^2) - \left(x^2 + \frac{x^{10}}{4}\right)\right] dx$$

$$= \int_{-2}^{2} \left(4 + 63x^2 - \frac{x^{10}}{4}\right) dx = 4x + 21x^3 - \frac{x^{11}}{44}\Big|_{-2}^{2}$$

$$= \left(8 + 168 - \frac{2^{11}}{44}\right) - \left(-8 - 168 - \frac{(-2)^{11}}{44}\right)$$

$$= 16 + 336 - \frac{2^{12}}{44} = 258.909\ldots$$