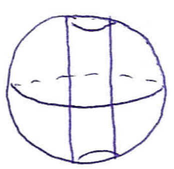
6) Use polar coordinates to **set up** the integral for the volume of the solid **inside both** the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 1$



Symmetrical: I will find the volume of the top half and double it

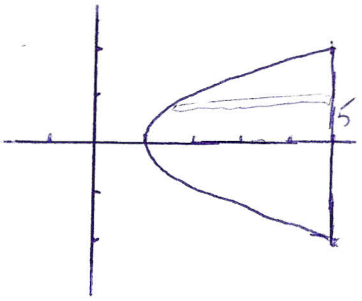
height is $z = \sqrt{4 - x^2 - y^2}$

Base on x-y plane $x^2 + y^2 \le 1$.

$$2 \cdot \iint_D \sqrt{4 - x^2 - y^2}\, dA$$

$$= 2 \int_0^{2\pi} \int_0^1 \sqrt{4 - r^2} \cdot r\, dr\, d\theta$$

7) Find expressions in terms of double integrals representing the mass and center of mass of the lamina that has density function ρ(x,y) and which occupies the region D where D is bounded by $x = y^2 + 1$ and $x = 5$



intersect

$y^2 + 1 = 5$

$y^2 = 4$

$y = \pm 2$

$$m = \int_{-2}^{2} \int_{y^2+1}^{5} \rho(x, y)\, dx\, dy$$

center of mass $\left(\frac{1}{m} \int_{-2}^{2} \int_{y^2+1}^{5} x\rho(x,y)\, dx\, dy,\ \frac{1}{m} \int_{-2}^{2} \int_{y^2+1}^{5} y\rho(x,y)\, dx\, dy \right)$

8) Evaluate

$$\int_0^1 \int_0^{2z} \int_0^{\ln x} x^3 e^y\, dy\, dx\, dz$$

$$\int_0^1 \int_0^{2z} x^3 e^y \Big|_{y=0}^{y=\ln x} dx\, dz = \int_0^1 \int_0^{2z} \left(x^3 e^{\ln x} - x^3 e^0 \right) dx\, dz$$

$$= \int_0^1 \int_0^{2z} (x^4 - x^3)\, dx\, dz = \int_0^1 \frac{x^5}{5} - \frac{x^4}{4} \Big|_0^{2z} dz = \int_0^1 \left(\frac{(2z)^5}{5} - \frac{(2z)^4}{4} \right) dz$$

$$= \int_0^1 \left(\frac{32}{5} z^5 - 4z^4 \right) dz = \frac{32}{30} z^6 - \frac{4z^5}{5} \Big|_0^1 = \frac{16}{15} - \frac{4}{5} = \frac{16}{15} - \frac{12}{15}$$

$$= \frac{4}{15}$$