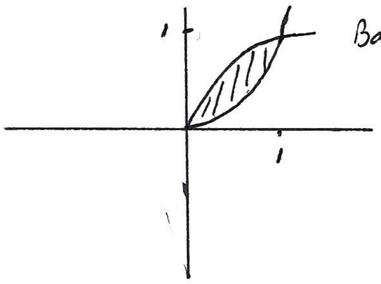
Math 251
Exam IV
Spring 2013

Show All Work

name Key

1.) **Set up** $\iiint_E x\,dV$ where E is bounded by the parabolic cylinders $y = x^2$ and $x = y^2$ and the planes $z = 0$ and $z = x + y$



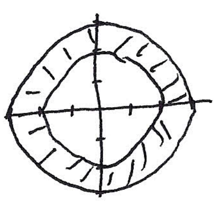
Base D

$$\iint_D \left[\int_0^{x+y} x\,dz\right] dA$$

$$= \int_0^1 \int_{x^2}^{\sqrt{x}} \int_0^{x+y} x\,dz\,dy\,dx$$

2.) Find $\iiint_E y^2\,dV$ where E is between the cylinders $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$, above the plane $z = 0$, and below the cone $z^2 = 4x^2 + 4y^2$.

Base D



$$\iint_D \left[\int_0^{\sqrt{4x^2+4y^2}} y^2\,dz\,dA\right]$$

use polar/cylindrical

$$\int_0^{2\pi}\int_2^3\int_0^{\sqrt{4r^2}} (r\sin\theta)^2\,dz \cdot r\,dr\,d\theta$$

note $\sqrt{4r^2} = 2r$

$$= \int_0^{2\pi}\int_2^3\int_0^{2r} r^3\sin^2\theta\,dz\,dr\,d\theta = \int_0^{2\pi}\int_2^3 z r^3\sin^2\theta\Big|_{z=0}^{2r} dr\,d\theta$$

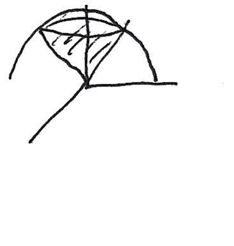
$$= \int_0^{2\pi}\int_2^3 2r^4\sin^2\theta\,dr\,d\theta = \int_0^{2\pi} \frac{2r^5}{5}\sin^2\theta\Big|_{r=2}^{3} d\theta = \int_0^{2\pi} \frac{486-64}{5}\cdot\sin^2\theta\,d\theta$$

$$= \frac{422}{5}\int_0^{2\pi}\sin^2\theta\,d\theta = \frac{422}{10}\int_0^{2\pi}(1-\cos 2\theta)\,d\theta = \frac{422}{10}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{422}{10}\left[(2\pi-0)-(0-0)\right] = \frac{422\pi}{5}$$

3.) **Set up** $\iiint_H xyz\,dV$ where E lies below the sphere $x^2+y^2+z^2 = 4$, and above the cone $z = \sqrt{x^2+y^2}$

spherical coordinates



$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{2} (\rho\sin\varphi\cos\theta)(\rho\sin\varphi\sin\theta)(\rho\cos\varphi)\cdot\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$