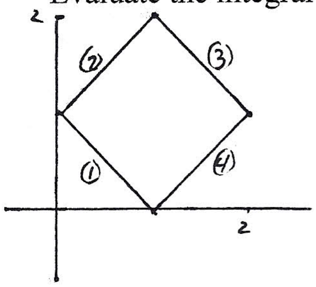
4.) Let R be the region bounded by the square with vertices (0, 1), (1, 2), (2, 1), and (1, 0). Evaluate the integral $\iint_R (x + y)\sin(x - y)\,dA$ by letting u = x + y and v = x – y.




① $y = -x+1 \rightarrow x+y=1 \rightarrow u = 1$

② $y = x+1 \rightarrow x-y=-1 \rightarrow v=-1$

③ $y = -x+3 \rightarrow y+x = 3 \rightarrow u=3$

④ $y = x-1 \rightarrow x-y=1 \rightarrow v=1$

$u = x+y$
$v = x-y$

ADDING $u+v = 2x$, $\frac{u+v}{2} = x$

subtracting $u-v = 2y$, $\frac{u-v}{2} = y$

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$

we need $\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \frac{1}{2}$

$$\int_{-1}^{1}\int_{1}^{3} u \sin v \cdot \frac{1}{2}\,du\,dv = \frac{1}{2}\int_{-1}^{1}\int_{1}^{3} u \sin v\,du\,dv = \frac{1}{2}\int_{-1}^{1} \frac{u^2}{2}\sin v\Big|_{u=1}^{u=3} dv$$

$$= \frac{1}{2}\int_{-1}^{1}\left(\frac{9}{2}-\frac{1}{2}\right)\sin v\,dv = 2\int_{-1}^{1}\sin v\,dv$$

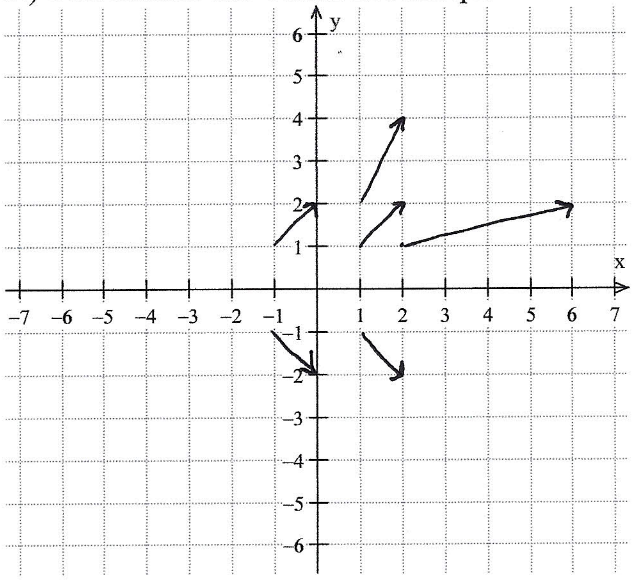
( you can note this is zero since its odd or finish

$$= -2\cos v\Big|_{-1}^{1} = -2\cos 1 + 2\cos(-1)$$

but $\cos(-1) = \cos 1$

$$= -2\cos 1 + 2\cos 1 = 0$$

5.) Plot at least one vector in each quadrant for the vector field **F** = < $x^2$, y>




| PT | F |
|---|---|
| (1, 1) | <1, 1> |
| (-1, 1) | <1, 1> |
| (-1, -1) | <1, -1> |
| (1, -1) | <1, -1> |
| more ? | |
| (1, 2) | <1, 2> |
| (2, 1) | <4, 1> |