6.) Evaluate the following line integrals.

a) $\int_C e^y\,dx + x\,dy$ where C is the part of the curve y = lnx from (1, 0) to (e, 1).

TWO MOST OBVIOUS PARAMETRIZATIONS GIVEN.

① $x = t \quad y = \ln t \quad 1 \le t \le e$

$$\int_1^e e^{\ln t}\frac{dx}{dt}\,dt + t\frac{dy}{dt}\,dt = \int_1^e t\cdot 1\cdot dt + t\cdot\frac{1}{t}\cdot dt = \int_1^e (t+1)\,dt$$

$$= \frac{t^2}{2}+t\Big|_1^e = \left(\frac{e^2}{2}+e\right)-\left(\frac{1}{2}+1\right) = \frac{e^2}{2}+e-\frac{3}{2}$$

② $y = t \quad x = e^t \quad 0 \le t \le 1$

$$\int_0^1 e^t\cdot\frac{dx}{dt}\,dt + e^t\cdot\frac{dy}{dt}\,dt = \int_0^1 e^t\cdot e^t\,dt + e^t\cdot 1\,dt$$

$$= \int_0^1 (e^{2t}+e^t)\,dt = \frac{e^{2t}}{2}+e^t\Big|_0^1 = \left(\frac{e^2}{2}+e\right)-\left(\frac{1}{2}+1\right)$$

$$= \frac{e^2}{2}+e-\frac{3}{2}$$

b) $\int_C (x+y+z)\,ds$ where C is the line segment from (1, 2, 3) to (5, 1, 4). direction $\langle 4,-1,1\rangle$

$\vec{r}(t) = \langle 4t+1, -t+2, t+3\rangle \quad 0 \le t \le 1 \quad \vec{r}\,'(t) = \langle 4,-1,1\rangle$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2} = \sqrt{4^2+(-1)^2+(1)^2} = \sqrt{18} = 3\sqrt{2}$$

$$\int_C (x+y+z)\,ds = \int_0^1 [(4t+1)+(-t+2)+(t+3)]\cdot 3\sqrt{2}\,dt$$

$$= \int_0^1 (4t+6)\cdot 3\sqrt{2}\,dt = 6\sqrt{2}\int_0^1 (2t+3)\,dt = 6\sqrt{2}\left[t^2+3t\right]_0^1$$

$$= 6\sqrt{2}\left[(1+3)-(0+0)\right] = 24\sqrt{2}$$

7.) A)Determine whether the function **F**(x, y) = <y^2, 2xy – e^y> is conservative. If it is, find its potential function.

$$\frac{dQ}{dx} = 2y = \frac{dP}{dy}$$ – yes its conservative

$$f(x,y) = y^2x - e^y + K$$