8.) Let $\mathbf{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$.

Under what conditions is $\int_C \mathbf{F} \cdot d\mathbf{r} = \int_c P(x,y)dx + Q(x,y)dy$ independent of path?

SOME ACCEPTABLE ANSWERS

1) $\vec{F}$ IS CONSERVATIVE

2) $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$

9.) Find the work done by $\mathbf{F}(x, y) = \langle 9x^2y^2, 6x^3y - 1 \rangle$ in moving an object from (0,0) to (5,9).

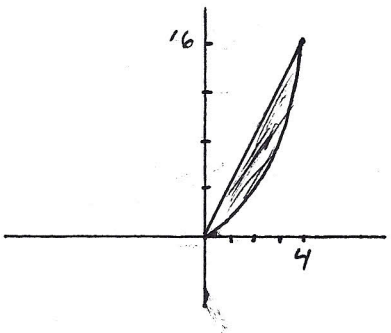
no path given but $\frac{\partial Q}{\partial x} = 18x^2y = \frac{\partial P}{\partial y}$ its conservative and independent of path

potential function $f(x,y) = 3x^3y^2 - y + K$

$$W = \int_C \vec{F} \cdot d\vec{r} = 3x^3y^2 - y \Big|_{(0,0)}^{(5,9)} = [(3 \cdot 125 \cdot 81) - 9] - 0$$

$$= 30375 - 9 = 30366$$

10.) Find $\int_C y^2\,dx + x^2\,dy$ where C is the boundary of the region lying between the graphs of $y = 4x$ and $y = x^2$



intersect $4x = x^2$

$0 = x^2 - 4x$

$0 = x(x-4)$

$x = 0$ or $x = 4$

simple closed

GREEN'S THEOREM

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \int_0^4 \int_{x^2}^{4x} (2x - 2y)\,dy\,dx$$

$$= \int_0^4 (2xy - y^2) \Big|_{x^2}^{4x} dx = \int_0^4 \left[(8x^2 - 16x^2) - (2x^3 - x^4)\right] dx$$

$$= \int_0^4 (x^4 - 2x^3 - 8x^2)\,dx = \frac{x^5}{5} - \frac{x^4}{2} - \frac{8x^3}{3} \Big|_0^4$$

$$= \left(\frac{1024}{5} - 128 - \frac{512}{3}\right) - (0) = \frac{3072 - 1920 - 2560}{15} = \frac{-1408}{15}$$