Math 441 Name____________________

Exam II

Fall 2011

Show All Work

1) Find the determinant of $\begin{matrix} 2 & 3 & 5 & 5 & 2 & 2 \\ 4 & 0 & 2 & 0 & 3 & 0 \\ 6 & 0 & 1 & 0 & 1 & 0 \\ 2 & 3 & 2 & 4 & 5 & 0 \\ 2 & 0 & 3 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{matrix}$

$$\begin{matrix} 2 & 3 & 5 & 5 & 2 & 2 \\ 4 & 0 & 2 & 0 & 3 & 0 \\ 6 & 0 & 1 & 0 & 1 & 0 \\ 2 & 3 & 2 & 4 & 5 & 0 \\ 2 & 0 & 3 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{matrix} = -2 \cdot \begin{matrix} 4 & 0 & 2 & 0 & 3 \\ 6 & 0 & 1 & 0 & 1 \\ 2 & 3 & 2 & 4 & 5 \\ 2 & 0 & 3 & 3 & 4 \\ 0 & 0 & 0 & 0 & 2 \end{matrix} = -4 \cdot \begin{matrix} 4 & 0 & 2 & 0 \\ 6 & 0 & 1 & 0 \\ 2 & 3 & 2 & 4 \\ 2 & 0 & 3 & 3 \end{matrix}$$

$$= 12 \cdot \begin{matrix} 4 & 2 & 0 \\ 6 & 1 & 0 \\ 2 & 3 & 3 \end{matrix} = 36 \cdot \begin{matrix} 4 & 2 \\ 6 & 1 \end{matrix} = 36 \cdot\ 4 - 12 \ = -288$$

2) Let A be a singular matrix. Show that it is always true that AB is a singular matrix.

Since A is singular we know that |A| = 0. This implies that |AB| = |A||B|= 0·|B|= 0.
But since |AB| = 0, we know AB must be singular.

3) Let **A** and **B** be 2x2 matrices where |**A**| = 5, |**B**| = -3, and $\mathbf{A}^t$ is the transpose of **A**. Find the following or state "can't be found" from the information given.

a) $|\mathbf{A}^{-1}|$ = 1/5

b) | A**B**| = -15

c) | A + **B**| = "can't be found"

d) $|\mathbf{B}^2|$ = 9

e) |3**A**| = 45

f) $|\mathbf{A}^t|$ **= 5**

g) If **C** is the matrix that results when one row two rows of matrix **A** (above) have been switched then | **C**| = -|A| = -5

True or false

a. The set of all polynomials of degree 2 forms a vector space. False (If you don't include polynomials of smaller degree it is not closed under addition)

b. In theory, Cramer's Rule can be used to find all solutions of any linear system that has the same number of variables as equations. False. (Cramer's Rule only applies if the determinant of the coefficient matrix is not zero).

5) Given

$$\begin{aligned} x - 5y + 4z &= -3 \\ 2x - 7y + 2z &= -3 \\ -2x + y + 9z &= -1 \end{aligned}$$

Cramers rule says y = ???? (Note: you do not have to solve)

$$\frac{\begin{matrix} 1 & -3 & 4 \\ 2 & -3 & 2 \\ -2 & -1 & 9 \end{matrix}}{\begin{matrix} 1 & -5 & 4 \\ 2 & -7 & 2 \\ -2 & 1 & 9 \end{matrix}}$$

6) a) Find the adjoint of $\mathbf{A} = \begin{matrix} 2 & 0 & 5 \\ 1 & 0 & 3 \\ 0 & 1 & 0 \end{matrix}$

Matrix of Minors $\begin{matrix} -3 & 0 & 1 \\ -5 & 0 & 2 \\ 0 & 1 & 0 \end{matrix}$, Cofactor matrix $\begin{matrix} -3 & 0 & 1 \\ 5 & 0 & -2 \\ 0 & -1 & 0 \end{matrix}$

Adjoint $\begin{matrix} -3 & 5 & 0 \\ 0 & 0 & -1 \\ 1 & -2 & 0 \end{matrix}$

b) USE the ADJOINT above to find the inverse of A.

$A^{-1} = = \frac{1}{A} \cdot Adj\ A = \begin{matrix} -3 & 5 & 0 \\ 0 & 0 & -1 \\ 1 & -2 & 0 \end{matrix}$

… scratch work….. $|A| = 1 \cdot \begin{matrix} -3 & 5 \\ 1 & -2 \end{matrix} = 1$

7) Let V be a set on which two operations (addition and scalar multiplication) are defined. What 10 axioms must be satisfied for every u, v, and w in V, and every scalar c and d, before we can call V a vector space?

i. **u + v** ∈ V
ii. u + v = v + u
iii. u + (v + w) = (u + v) + w
iv. There is an element **0** such that
u + 0 = 0 + u = u
v. There is an element –**u** such that
u + -u = -u + u = 0.
vi. C**u** ∈ V
vii. (c + d)**u** = c**u** + d**u**
viii. c(**u + v**) = c**u** + c**v**
ix. (cd)**u** = c(d**u**)
x. 1·**u** = **u**

8) If possible write **v** = (7, 4, 9) as a linear combination of $\mathbf{u_1}$ = (1, 1, 2), $\mathbf{u_2}$ = (1, 1, 1), and $\mathbf{u_3}$ = (2, 1, 1) .

$$\begin{matrix} 1 & 1 & 2 & 7 \\ 1 & 1 & 1 & 4 \\ 2 & 1 & 1 & 9 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & 2 & 7 \\ 0 & 0 & -1 & -3 \\ 0 & -1 & -3 & -5 \end{matrix} \rightarrow \begin{matrix} 1 & 1 & 2 & 7 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & -1 & -3 \end{matrix} \rightarrow$$

$$\begin{matrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & -1 & -3 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 3 \end{matrix} \rightarrow \begin{matrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 3 \end{matrix}$$

Thus **v** = $5\mathbf{u_1}$-$4\mathbf{u_2}$+$3\mathbf{u_3}$

9) Why does $\mathbf{R}^2$ under the operations defined below fail to be a vector space? Identify at least one axiom that fails.

Addition: $(x_1, y_1) + (x_2, y_2) = (x_1 * x_2, y_1 * y_2)$
Scalar Multiplication: c(x, y) = (cx, cy)

$c*\{ (x_1, y_1) + (x_2, y_2)\}= c\ (x_1 * x_2, y_1 * y_2) = (c*x_1 * x_2, c*y_1 * y_2)$

However,
$c(x_1, y_1) + c(x_2, y_2) = (c*x_1, c*y_1) + (c*x_2, c*y_2) = (c^2*x_1*x_2, c^2*y_1*y_2)$

These are not the same. The rule falis.

10) a) Let W = {(x,y): $x \geq 0$, $y \geq 0$} with the standard operations in R^2. Does W form a subspace of R^2? Why or why not? SHOW WORK.

NO. It is not closed under scalar multiplication. If $(x, y) \in S$ then $-2(x, y) \notin S$

11) Given a set of vectors S = { v_1, v_2, ….v_k} in a vector space V. If the vector equation $c_1v_1 + c_2v_2 + \ldots + c_kv_k = 0$ has only the trivial solution then S is called…

a. linearly independent ← correct answer

~~b. linearly dependent~~

~~c. a spanning set of V~~

~~d. singular~~

~~e. homogeneous~~

12) Does S = {(1, 2, 3), (1, 1, 1), (4, 3, 1)} span R^3? Give a reason for your answer

$$\begin{matrix} 1 & 1 & 4 \\ 2 & 1 & 3 \\ 3 & 1 & 1 \end{matrix} = \begin{matrix} 1 & 1 & 4 \\ 0 & -1 & -5 \\ 0 & -2 & -11 \end{matrix} = \begin{matrix} -1 & -5 \\ -2 & -11 \end{matrix} = 11 - 10 = 1$$

Yes, since the determinant is not zero the matrix row reduces to I. This means that there is always a solution to the system $c_1(1, 2, 3) + c_2(1,1,1) + c_3(4, 3, 1) = (x, y, z)$

13) Is S = {x, x+1, $x^2 + 1$} is linearly independent in P_2. Give a reason for your answer.

Reminder:

$ax + b(x+1) + c(x^2 + 1) = \mathbf{0}$

$b + c + ax + bx + cx^2 = \mathbf{0}$

means

$$\begin{aligned} b + c &= 0 \\ a + b &= 0 \\ c &= 0 \end{aligned}$$

$$\begin{matrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} = -1 \cdot \begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix} = -1$$

Since the determinant of the coefficient matrix is not zero it row reduces to I. Thus the system has a unique solution. Therefor the vectors are linearly independent.