

- 3) (3 pts) Short answer: Let  $A$  be a  $5 \times 6$  matrix whose rank is  $k$   
 a) What is the largest value that  $k$  can be?

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- b) How many vectors are in a basis for the row space of  $A$ ?

$k$

- c) State why  $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1$  is not an inner product for  $\mathbf{u} = (u_1, u_2)$ ,  $\mathbf{v} = (v_1, v_2)$

$$\langle (0, 1), (0, 1) \rangle = 0$$

So  $\langle \vec{v}, \vec{v} \rangle = 0$  but  $\vec{v} \neq \vec{0}$   
 not an inner product

- 4) <sup>12</sup> (10 pts) Let  $B = \{(1, 0, 1), (0, 1, 1), (0, 0, 1)\}$  and  $B' = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$   
 Find the transition matrix from  $B$  to  $B'$ .

$$\left[ \begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array} \right] \xrightarrow{-R_2+R_1} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array} \right]$$

$$\xrightarrow{-R_3+R_2} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array} \right]$$

$$P^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$