5) If $[\mathbf{A}|\,\mathbf{b}]$ row reduces to $\left[\begin{array}{ccccc|c} 1 & 0 & 2 & 0 & 3 & 2 \\ 0 & 1 & 3 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 & 6 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

a) (12 pts) Write the solution to the system $\mathbf{Ax} = \mathbf{b}$ in the form $\mathbf{x} = \mathbf{x_h} + \mathbf{x_p}$, where $\mathbf{x_h}$ is the solution $\mathbf{Ax} = \mathbf{0}$ and $\mathbf{x_p}$ is a particular solution of $\mathbf{Ax} = \mathbf{b}$.

$x_5 = t$

$x_4 = 3 - 6t$

$x_3 = s$

$x_2 = 4 - 3s - 5t$

$x_1 = 2 - 3t - 2s$

$$X = \begin{bmatrix} 2 - 2s - 3t \\ 4 - 3s - 5t \\ s \\ 3 - 6t \\ t \end{bmatrix} = s\begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ -5 \\ 0 \\ -6 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

X_h X_p

b) (3 pts) Find a basis for the solution space of $\mathbf{Ax} = \mathbf{0}$.

$$\left\{\begin{bmatrix} -2 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -5 \\ 0 \\ -6 \\ 1 \end{bmatrix}\right\}$$

c) (3 pts) Find the dimension of the solution space of $Ax = 0$.

2

d) (3 pts) Find the rank of A.

3

e) (3 pts) Find the nullity of A.

2

f) (3 pts) Find a basis for the row space of A.

$\{(1,0,2,0,3), (0,1,3,0,5), (0,0,0,1,6)\}$