6) (~~6~~ 7 pts apiece) Consider P_2 as an inner product space, with the inner product given by $<p,q> = \int_0^1 p(x)q(x)dx$. Use $p(x) = 5x^2$ and $q(x) = 4$ to find the following:

a) $<p, q>$

$$\int_0^1 20x^2 = \frac{20x^3}{3}\Big|_0^1 = \frac{20}{3}$$

b) $\|p\| = \sqrt{<p,p>} = \sqrt{\int_0^1 25x^4\,dx} = \sqrt{5x^5\Big|_0^1} = \sqrt{5}$

c) The unit vector in the direction of p.

$$\frac{p}{\|p\|} = \frac{5x^2}{\sqrt{5}} = \sqrt{5}\,x^2$$

d) The cosine of the angle between p and q.

$$\|q\| = \sqrt{\int_0^1 16\,dx} = \sqrt{16x\Big|_0^1} = \sqrt{16} = 4$$

$$\cos\theta = \frac{<p,q>}{\|p\|\cdot\|q\|} = \frac{20/3}{\sqrt{5}\cdot 4} = \frac{20}{3\cdot\sqrt{5}\,4} = \frac{5}{3\sqrt{5}} = \frac{\sqrt{5}}{3}$$

7) (5 pts) Determine if the set of vectors in R^3 is orthogonal and/or orthonormal. If the set is only orthogonal, normalize the set to produce and orthonormal set.
{(2, -2, 1), (2, 1, -2), (1, 2, 2)}

a b c

$\langle a,b\rangle = 4-2-2 = 0$ $\qquad \|a\| = \sqrt{4+4+1} = 3$

$\langle a,c\rangle = 2-4+2 = 0$ $\qquad \|b\| = \sqrt{4+1+4} = 3$

$\langle b,c\rangle = 2+2-4=0$ $\qquad \|c\| = \sqrt{1+4+4} = 3$

orthogonal <u>not</u> orthonormal

an orthonormal set would be

$$\left\{\left(\tfrac{2}{3},-\tfrac{2}{3},\tfrac{1}{3}\right),\left(\tfrac{2}{3},\tfrac{1}{3},-\tfrac{2}{3}\right),\left(\tfrac{1}{3},\tfrac{2}{3},\tfrac{2}{3}\right)\right\}$$