Math 441
Exam IV
Fall 2011

Name Key

Show All Work

1) Let T be the linear transformation $T:\mathbf{R}^2 \to \mathbf{R}^3$ given by $T(\mathbf{v}) = \mathbf{A}\mathbf{v}$ then $\mathbf{A}$ is a 3×2 matrix.

2) Which of the following functions $T: \mathbf{R}^2 \to \mathbf{R}^2$ is **not** a linear transformation? Circle all that apply.

a) $T(x, y) = (x + 2y, 3x - 4y)$
b) $T(x, y) = (1/x, 1/y)$
c) $T(x, y) = (x , 0)$
d) $T(x, y) = (x + y, xy)$
e) $T(x, y) = (3x + 2, 7x + 1)$

3) Let T be a linear transformation from P_2 to P_2 such that $T(1) = x$, $T(x) = 1 + x$, and $T(x^2) = x + x^2$. Then $T(2 + 4x + 3x^2) =$

$$T(2+4x+3x^2) = 2T(1) + 4T(x) + 3T(x^2)$$
$$= 2 \cdot x + 4(1+x) + 3(x+x^2)$$
$$= 2x + 4 + 4x + 3x + 3x^2$$
$$= 4 + 9x + 3x^2$$

4)

a) (true or false) Any linear function of the form $f(x) = mx + b$ is a linear transformation from R into R.

False

b) If T is a linear transformation from R^3 to R^5 then rank(T) + nullity(T) = 3

c) If $T: \mathbf{R}^2 \to \mathbf{R}^2$ is given by $T(x,y) = (2x + y, x + 2y)$ then the image of (1,5) is (7, 11)

d) If $T: \mathbf{R}^2 \to \mathbf{R}^2$ is given by $T(x,y) = (2x + y, x + 2y)$ then the preimage of (4,5) is (1,2)

$2x+y=4$
$x+2y=5$

$$\left[\begin{array}{cc|c} 2 & 1 & 4 \\ 1 & 2 & 5 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 2 & 5 \\ 2 & 1 & 4 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & -3 & -6 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & 1 & 2 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$$

e) TRUE-FALSE
T is a linear transformation with ker(T) = 0 if and only if for all **u** and **v** in V, T(**u**) = T(**v**) implies **u** = **v**.

TRUE

5) Which of the vector space below are isomorphic to R^4? Circle all that apply.

a) $M_{2,2}$
b) $M_{3,1}$
c) P_4
d) P_3
e) C[0,4]