

- 6) The linear transformation T is defined by $T(x) = Ax$ where $A = \begin{bmatrix} 1 & 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Find a) $\ker(T)$, b) nullity (T), c) range(T), and d) rank(T).

Solution space

$$\begin{aligned} x_5 &= t \\ x_4 &= -2t \\ x_3 &= -4t \\ x_2 &= s \\ x_1 &= -3s - t \end{aligned}$$

$$\ker T = \left\{ \begin{bmatrix} -3s-t \\ s \\ -4t \\ -2t \\ t \end{bmatrix} = s \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ -4 \\ -2 \\ 1 \end{bmatrix} \mid s, t \in \mathbb{R} \right\}$$

$$\text{nullity}(T) = \dim(\ker T) = 2$$

Range T = column space of A

$$A^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 4 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 4 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Range } T = \left\{ a(1, 0, 0, 0) + b(0, 1, 0, 0) + c(0, 0, 1, 0) \mid a, b, c \in \mathbb{R} \right\}$$

$$\text{Rank } T = \dim(\text{Range } T) = 3$$