7) Let T: $R^3 \to R^2$ be given by T(x,y,z) = (x, y, 0). Describe the kernel (geometrically).

$\ker T = \{(0,0,z) \mid z \in \mathbb{R}\}$

$\rightarrow$ The z axis geometrically

8) Determine whether or not the linear transformation T(x, y) = (x + 3y, x + 2y) is invertible. If it is, find its inverse.

$\det A = \begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = -1$ invertible

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & -1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -2 & 3 \\ 0 & 1 & 1 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$$

$$T^{-1}(x,y) = (-2x+3y,\ x-y)$$

9) Find the standard matrix for $T = T_1 \circ T_2$ when T_1=(x, y) = (x - 2y, 2x + 3y) and T_2(x, y) = (y, x)

$A_1 = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ $\qquad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A = A_1 A_2 = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$