10) a) Find the standard matrix **A** for the linear transformation T(x, y) = (x +3y, x + 2y).

$$\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$$

b) If B' = {(1, 1), (-1, 0)} find the matrix **A'** of T relative to B'. (Do **not** assume similarity to do this step! Use the definition)

$$T(1,1) = (4,3) = a_1(1,1) + a_2(-1,0)$$

$$T(-1,0) = (-1,-1) = b_1(1,1) + b_2(-1,0)$$

$$\left[\begin{array}{cc|cc} 1 & -1 & 4 & -1 \\ 1 & 0 & 3 & -1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & -1 & 4 & -1 \\ 0 & 1 & -1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -1 & 0 \end{array}\right]$$

$$A' = \begin{bmatrix} 3 & -1 \\ -1 & 0 \end{bmatrix}$$

c) Find the appropriate transition matrices and show that **A'** is similar to **A**.

P: transition from B' to Standard

$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 \end{array}\right] \longrightarrow \quad P = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

P^{-1}: transition from standard to B'

$$\left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$$P^{-1}AP = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -1 & 0 \end{bmatrix}$$

$$= A'$$

Yipee!