## Announcements

- Exam schedule: Feb 11 (Clark 101), Mar 8, Apr 10 (both White B51), 5-7 PM
- Makeup schedule: Feb 12, Mar 5, Apr 9, 5-7 PM, now in Clark 317
- Final exam: May 1, 2 - 4 PM, White B51


## Example problem

A sled is tied to a tree on a frictionless (snow covered) hill. If the sled weighs 77 N , find the magnitude of the tension force exerted by the rope on the sled and that of the normal force exerted by the hill on the sled.


1st step: Draw the free body diagram. Which forces act on the sled (dark blue arrows)?


## Summary

- Newton's 3rd law: If object A exerts a force on object B (Action Force), object B will always exert a force on object A that is equal in magnitude, but opposite in direction (Reaction Force)


## Action $=-$ Reaction

- If an object is located on a surface at rest, at least two forces will act on it: (i) The gravitational force (downwards) and (ii) the normal force (upwards) exerted by the surface on the object to compensate gravitation.
- A free body diagram is a schematic that shows all forces acting on a given body (no other forces). Always draw such a diagram, when facing a problem related to forces.
- In problems about inclined planes, calculate all forces parallel and perpendicular to the inclined plane.



## What is friction?



- If we push or pull an object in contact with some medium (e.g. surface, air, water), a friction force will be present, that acts in the opposite direction as the applied force.
- The friction force results from an interaction between the object and the surrounding medium.
- There are two different types of friction:

1. Static friction (objects that do not move, $v=0 \mathrm{~m} / \mathrm{s}$ )
2. Kinetic friction (moving objects, $v \neq 0 \mathrm{~m} / \mathrm{s}$ )

## Static friction

- If a heavy object sits on a surface, I will not be able to move it by pushing it softly.


## Why not? - I am applying a force to one side. Why does the object not accelerate, i.e. start moving?

- There must be a force that compensates the externally applied force - this is the static friction force!

$$
\left|\vec{f}_{s}\right| \leq \mu_{s}|\vec{n}|
$$

- The static friction force compensates for the externally applied force until it reaches its max. value.
- This maximum value is determined by the static friction coefficient, $\mu_{\mathrm{s}}$, of both surface materials and the normal force.
- If the external force exceeds this max. value, the object will start to move.

For small applied forces, the magnitude of the force of static friction equals the magnitude of the applied force.


## Kinetic friction

- Once the object starts moving the friction force changes from static to kinetic friction:

$$
\left|\vec{f}_{k}\right|=\mu_{k}|\vec{n}|
$$

$\mu_{\mathrm{k}}$ is the kinetic friction coefficient, which is again determined by the two interacting surface materials.

- Kinetic friction causes moving objects to slow down, if no external force is applied (e.g. a moving car).
- The kinetic friction coefficient is always smaller than the static friction coefficient, i.e. it is easier to accelerate an object once it is moving:

$$
\mu_{k}<\mu_{s}
$$

When the magnitude of the applied force exceeds the magnitude of the maximum force of static friction, the trash can breaks free and accelerates to the right.


## Friction coefficients

Table 4.2 Coefficients of Frictiona ${ }^{\text {a }}$

|  | $\boldsymbol{\mu}_{\boldsymbol{s}}$ | $\boldsymbol{\mu}_{\boldsymbol{k}}$ |
| :--- | :---: | :---: |
| Steel on steel | 0.74 | 0.57 |
| Aluminum on steel | 0.61 | 0.47 |
| Copper on steel | 0.53 | 0.36 |
| Rubber on concrete | 1.0 | 0.8 |
| Wood on wood | $0.25-0.5$ | 0.2 |
| Glass on glass | 0.94 | 0.4 |
| Waxed wood on wet snow | 0.14 | 0.1 |
| Waxed wood on dry snow | - | 0.04 |
| Metal on metal (lubricated) | 0.15 | 0.06 |
| Ice on ice | 0.1 | 0.03 |
| Teflon on Teflon | 0.04 | 0.04 |
| Synovial joints in humans | 0.01 | 0.003 |

${ }^{a}$ All values are approximate.


No applied force, box at rest. No friction:


Weak applied force, box remains at rest.

Static friction:


Stronger applied force,
box just about to slide.
Static friction:


Box sliding at
constant speed.
Kinetic friction:


## Heavy vs. light objects

- Heavier objects are more difficult to move on a given surface than light objects, since the friction force is proportional to the normal force:

$$
\left|\vec{f}_{k}\right|=\mu_{k}|\vec{n}| \quad|\vec{n}|=m g
$$

The reason is the stronger normal force for heavy objects.

- Friction forces are independent of the area of contact between the surfaces.
- Friction is the reason why we do not slip, when a weak force is exerted on us.

For small applied forces, the magnitude of the force of static friction equals the magnitude of the applied force.


## Example problem

You want to move a $500-\mathrm{N}$ crate across a level floor. To start the crate moving, you have to pull with a $230-\mathrm{N}$ horizontal force. Once the crate "breaks loose" and starts to move, you can keep it moving at constant velocity with only 200 N . What are the coefficients of static and kinetic friction?
(a) Pulling a crate
(b) Free-body diagram for crate just before it starts to move

(c) Free-body diagram for crate moving at constant speed


## Before the crate moves

(b) Free-body diagram for crate just before it starts to move

$$
\begin{array}{ll}
\sum F_{x}=T+\left(-\left(f_{\mathrm{s}}\right)_{\max }\right)=0 & \text { so } \quad\left(f_{\mathrm{s}}\right)_{\max }=T=230 \mathrm{~N} \\
\sum F_{y}=n+(-w)=0 & \text { so } \quad n=w=500 \mathrm{~N}
\end{array}
$$

$$
f_{s}=\mu_{s} n
$$

$$
\mu_{\mathrm{s}}=\frac{\left(f_{\mathrm{s}}\right)_{\max }}{n}=\frac{230 \mathrm{~N}}{500 \mathrm{~N}}=0.46
$$

## After the crate starts moving

$$
\begin{array}{ll}
\sum F_{x}=T+\left(-f_{\mathrm{k}}\right)=0 & \text { so } \quad f_{\mathrm{k}}=T=200 \mathrm{~N} \\
\sum F_{y}=n+(-w)=0 & \text { so } \quad n=w=500 \mathrm{~N}
\end{array}
$$

$$
f_{k}=\mu_{k} n
$$

$$
\mu_{\mathrm{k}}=\frac{f_{\mathrm{k}}}{n}=\frac{200 \mathrm{~N}}{500 \mathrm{~N}}=0.40
$$

$$
W=500 \mathrm{~N}
$$

## Friction force depends on object's velocity



In reality, the friction force is proportional to the object's velocity. We neglect this.
Thus, the speed of a skydiver does not increase continuously after jumping out of a plane, but reaches a terminal speed, for which the air friction force compensate gravitation.

## Summary

- Friction forces result from the interaction of an object with its surrounding medium and are directed into the opposite direction of an externally applied force.
- There two different types of friction forces:
(i) Static friction (resting objects):

$$
\left|\overrightarrow{f_{s}}\right| \leq \mu_{s}|\vec{n}|
$$

(ii) kinetic friction (moving objects):

$$
\left|\overrightarrow{f_{k}}\right|=\mu_{k}|\vec{n}|
$$

- Friction forces are proportional to the normal
 force on an object, i.e. heavy objects are more difficult to move.
- Friction forces are determined by friction coefficients. The static friction coefficient is always higher than the kinetic friction coefficient: $\quad \mu_{k}<\mu_{s}$


## More review problems

## Free Fall example problem

A baseball is thrown up in the air at an initial velocity of $22.0 \mathrm{~m} / \mathrm{s}$.
(a) How high up does it go?
(b) How long is it in the air if you catch it at the same height you initially let go of the ball?


Some helpful equations:

$$
\begin{aligned}
& v(t)=v_{0}-g t \\
& y(t)=y_{0}+v_{0} t-\frac{1}{2} g t^{2}
\end{aligned}
$$

## Projectile motion example problem

A fireman $\mathrm{d}=57 \mathrm{~m}$ away from a burning building directs a stream of water from a ground-level fire hose at an angle of 23 . above the horizontal.

If the speed of the stream as it leaves the hose is $\mathrm{v}_{\mathrm{i}}=40 \mathrm{~m} / \mathrm{s}$, at what height will the stream of water strike the building?


## Average and instantaneous velocity



What is the average velocity in the time interval between 1 s and 3 s ?
What is the instantaneous velocity at $\mathrm{t}=1 \mathrm{~s}$ ?

## Newton's 2nd law



A boat moves through the water with two forces acting on it. One is a 2000 N forward push by the water on the propeller, and the other is a 1800 N resistive force due to the water around the bow.
A. What is the acceleration of the 1000 kg boat?
B. If it starts from rest, how far will the boat move in 10s?
C. What will its velocity be at the end of that time?

## The bird feeder

A 150 N bird feeder is supported by three cables.
Find the tension in each cable.


## Weighing a fish in the elevator

A woman weighs a fish with a spring scale attached to the ceiling of an elevator. While the elevator is at rest, she measures a weight of 40 N .
A. What weight does the scale read if the elevator accelerates upwards at $2 \mathrm{~m} / \mathrm{s}^{2}$ ?
B. What does the scale read if the elevator accelerates downwards at $2 \mathrm{~m} / \mathrm{s}^{2}$ ?

When the elevator accelerates upward, the spring scale reads a value greater than the weight of the fish.


## Example problem: Friction

A hockey puck struck by a hockey stick is given an initial speed $v_{0}$ in the positive $x$-direction. The coefficient of kinetic friction between the ice and the puck is $\mu_{\mathrm{k}}$.
A. Obtain an expression for the acceleration of the puck as a function of $\mu_{\mathrm{k}}$ and g .
B. Use the result of part (a) to obtain an expression for the distance, d , the puck slides. The answer should be in terms of $v_{0}, \mu_{k}$, and $g$ only.


## The runaway car



b

A car of mass $m$ is on an icy driveway inclined at an angle of $20^{\circ}$.
A. Determine the acceleration of the car (no friction)
B. If the length of the driveway is 25 m and the car starts from rest at the top, how long does it take to travel to the bottom?
C. What is the car's speed at the bottom?

