Flame Acceleration due to Wall Friction: Accuracy and Intrinsic Limitations of the Analytical Formulation

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Motivation & Objectives

- Due to wall friction, propagation speeds of flames in tubes/channels can grow by several orders of magnitude. Such a *flame acceleration* may subsequently result in a deflgration-to-detonation transition (DDT). The DDT stays behind countless disasters in mines and power plants; and it can be utilized, constructively, in novel energy efficient setups such as pulse-detonation engines.
- For decades, there was a limited theoretical understanding of the DDT mechanism because of the common opinion that flame acceleration is impossible without *turbulence*: the lack of knowledge about turbulence and turbulent flames prevented a rigorous DDT formulation to be developed.
- It was next realized that turbulence plays a *supplementary* role in the acceleration scenario such that even laminar flames can accelerate and initiate detonation due to wall friction [1,2].
- Based on this constructive idea, conceptually-laminar, rigorous formulations to quantify the flame acceleration scenario in channels and tubes have eventually been developed and validated [3,4].
- However, the formulations [3,4] employ a set of assumptions; this thereby leads to the intrinsic limitations of the formulations, which have not been properly identified so far.
- Identification of the intrinsic limitations and accuracy of the formulations [3,4] and quantification of their validity domains constitute the overall goal of the present work.

Analytical Formulations of Flame Acceleration in Channels/Tubes

- The formulations [3,4] are based on the following approximations: *(i) zero flame thickness; (ii) incompressible, near-isobaric combustion process; (iii) plane parallel flame generated flow in the fuel mixture;*

Here U_w is the total burning rate, and S_L the normal flame velocity

- The average flame-generated flow velocity is related to the total burning rate as: $\langle u_z\rangle = (\Theta-1)U_w.$
- The exponential state of the flame acceleration is exhibited $U_w \propto \exp(\sigma S_L t/R).$
- Flame evaluation equation: $w_z(0,\tau)-w_z(\eta,\tau)=\sqrt{1+\left(\frac{\partial f}{\partial\eta}\right)^2}-1-\frac{\partial f}{\partial\tau}\approx\frac{\partial f}{\partial\eta}-\frac{\partial f}{\partial\tau}$
- Plane-parallel Navier-Stokes equation, 2D and cylindrical-axisymmetric

$$\frac{\partial w_z}{\partial\tau}=-\frac{\partial p}{\partial\xi}+\frac{1}{\mathrm{Re}}\frac{\partial^2 w_z}{\partial\eta^2},$$

$$\frac{\partial w_z}{\partial\tau}=-\frac{\partial p}{\partial\xi}+\frac{1}{\mathrm{Re}}\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial w_z}{\partial\eta}\right).$$

Flame Acceleration in 2-D channels

- The major result of the 2D formulation [3] is a coupling of the flame acceleration rate σ to the thermal expansion ratio $\Theta=\rho_f/\rho_b$ and a flame propagation Reynolds number $\mathrm{Re}=RS_L/\nu$,

$$\frac{\mu\cosh\mu-\sinh\mu}{\mu(\Theta-1)}=\frac{\exp\mu}{2(\mu+\sigma)}-\frac{\exp(-\mu)}{2(\mu-\sigma)}+\frac{\mu^2}{\mu^2-\sigma^2}\frac{\exp(-\sigma)}{\sigma}-\frac{1}{\sigma}\qquad \mu=\sqrt{\sigma\,\mathrm{Re}},$$

- This equation can be solved analytically in the limit of $\mu>>1$, *i.e.* $\Theta>>1$ with the acceleration rate

$$\sigma=\frac{(\mathrm{Re}-1)^2}{4\mathrm{Re}}\left(\sqrt{1+\frac{4\mathrm{Re}\,\Theta}{(\mathrm{Re}-1)^2}}-1\right)^2$$

Flame Acceleration in Cylindrical Tubes

- The major result of the formulation [4], for the cylindrical-axisymmetric coordinates, is the equation for the acceleration rate σ:

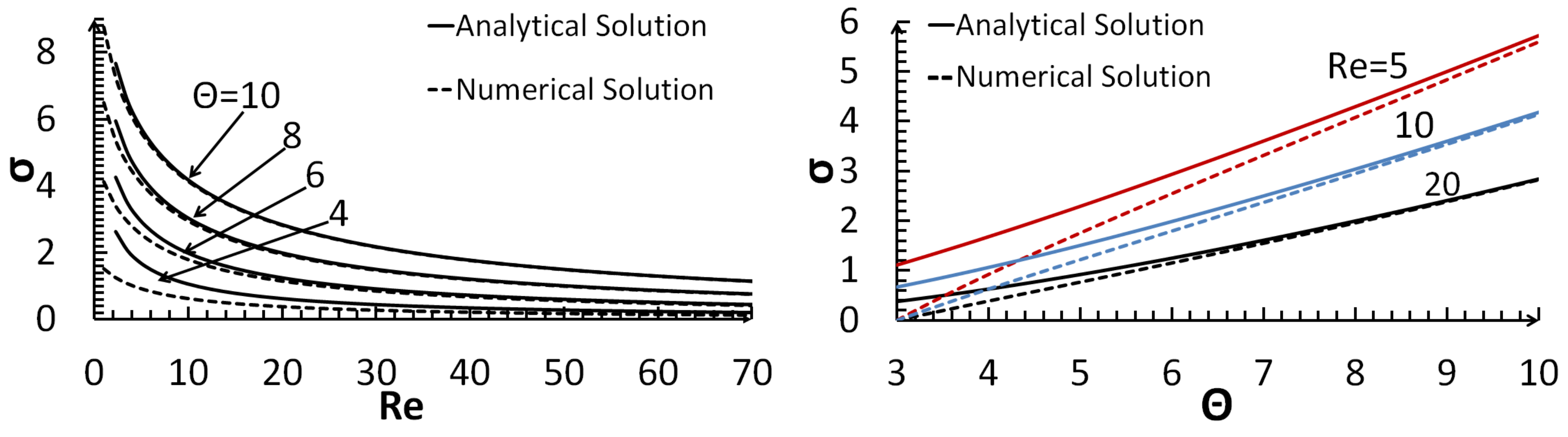
$$\frac{I_0(\mu)-2\mu^{-1}I_1(\mu)}{2(\Theta-1)}=\frac{(\sigma+1)\exp(-\sigma)-1}{\sigma^2}+\Psi(1)\exp(-\sigma)-\int_0^1\Psi(\eta)\exp(-\sigma\eta)d\eta,\qquad \Psi(\eta)=\int_0^\eta I_0(\mu\chi)\exp(\sigma\chi)d\chi$$

- Within the 0th- and 1st-order approximations in $\mu^{-1}\ll 1$, this equation respectively yields the asymptotic result

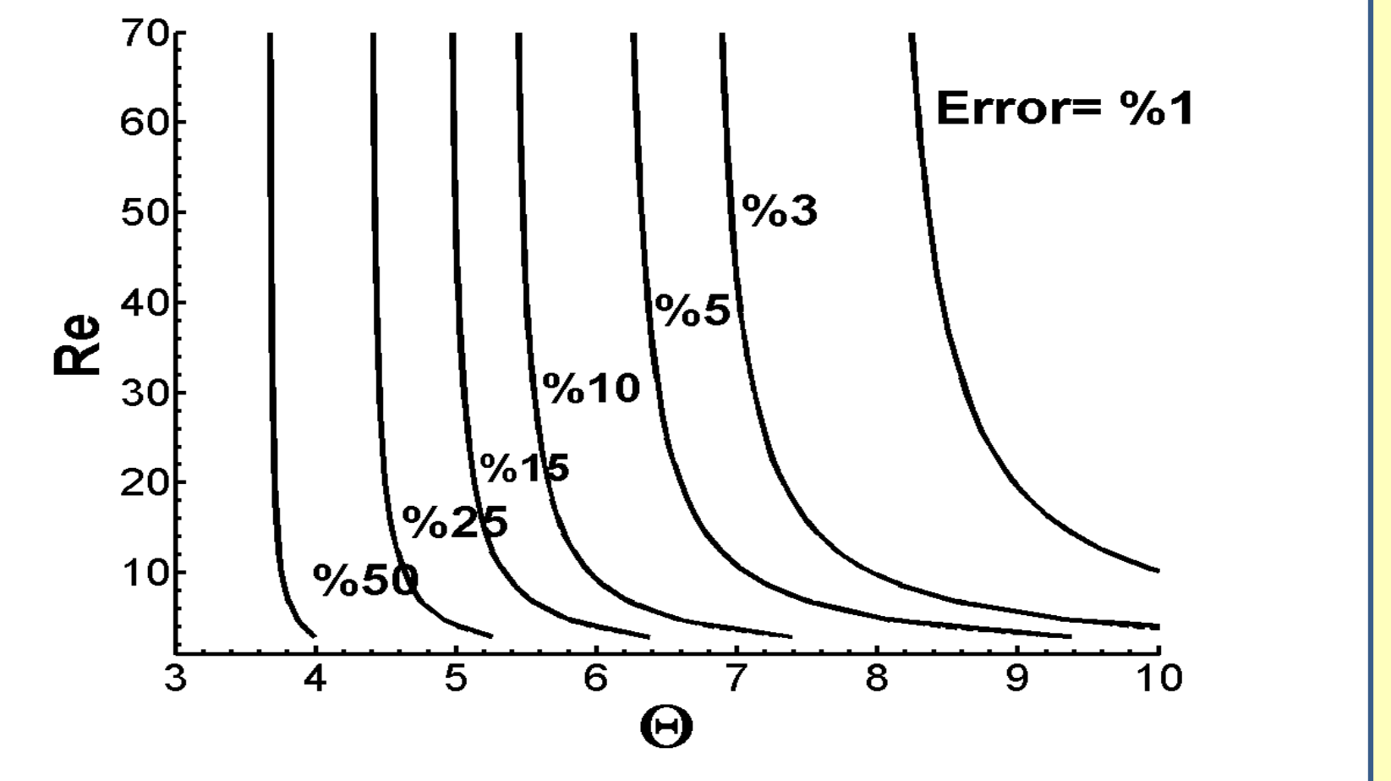
$$\mu_0=\sqrt{\sigma_0\,\mathrm{Re}},\qquad \sigma_0=\frac{\mathrm{Re}}{4}\left(\sqrt{1+\frac{8(\Theta-1)}{\mathrm{Re}}}-1\right)^2$$

$$\sigma_1=\frac{\mathrm{Re}}{4}\left(\sqrt{1+\frac{8(\Theta-1)}{\mathrm{Re}}\left[1+\frac{1}{\mu_0}+\frac{1}{2(\mu_0+\sigma_0)}\right]}-1\right)$$

2D Results

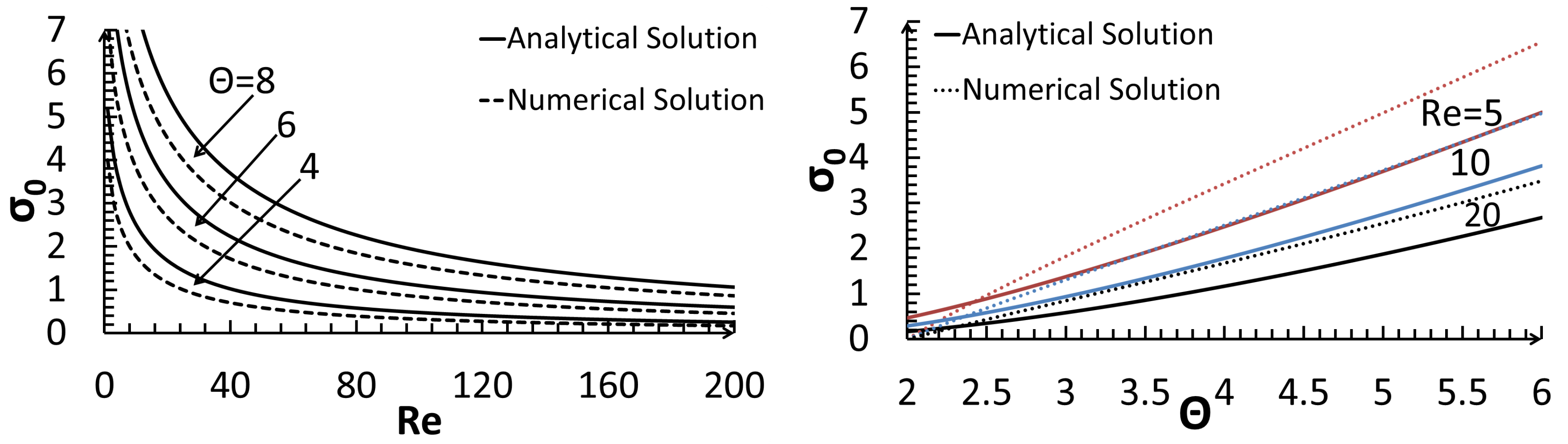


The flame acceleration rate σ vs the flame propagation Reynolds number Re at the fixed thermal expansion ratio Θ (left); and σ vs Θ at fixed Re (right).



Contour scheme (the error isolines (in %)) demonstration is above in the Re-Θ diagram

Cylindrical Results



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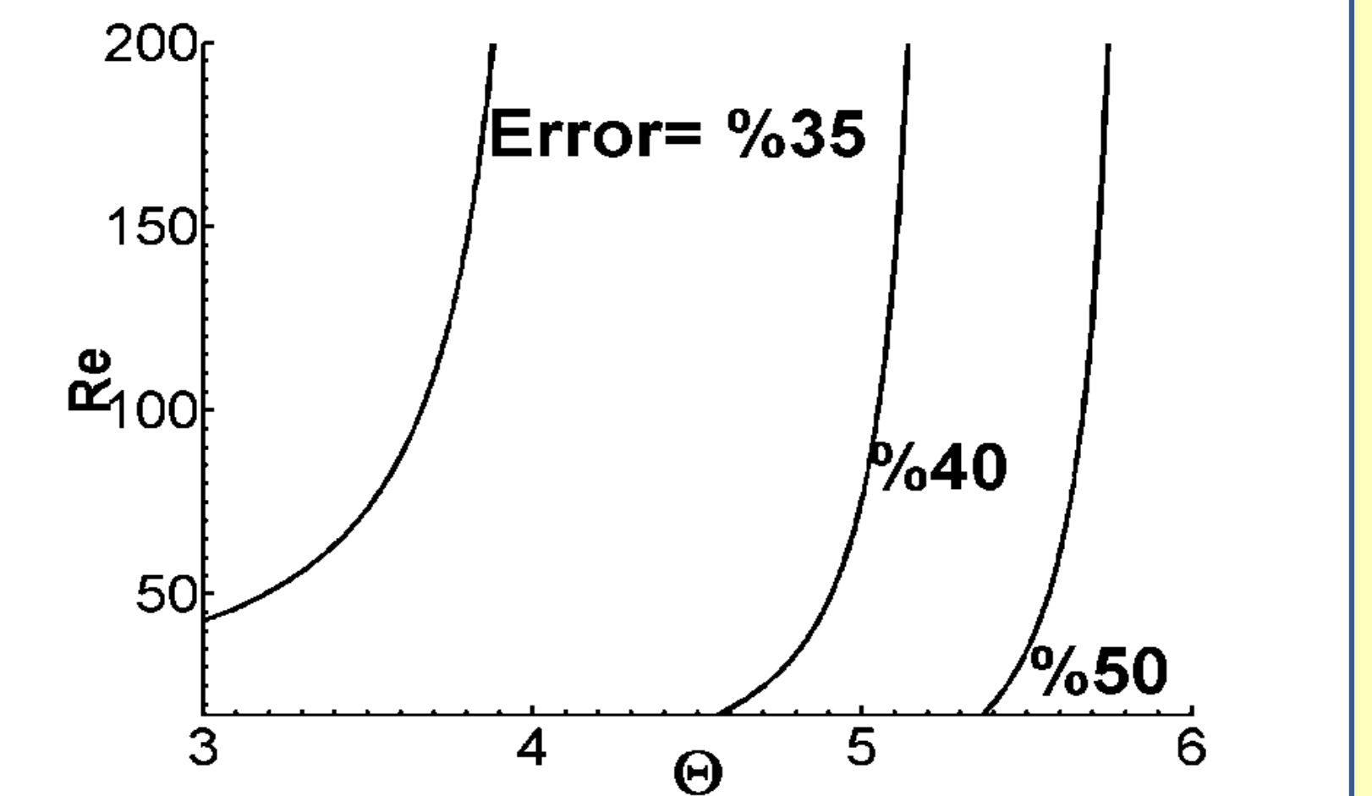

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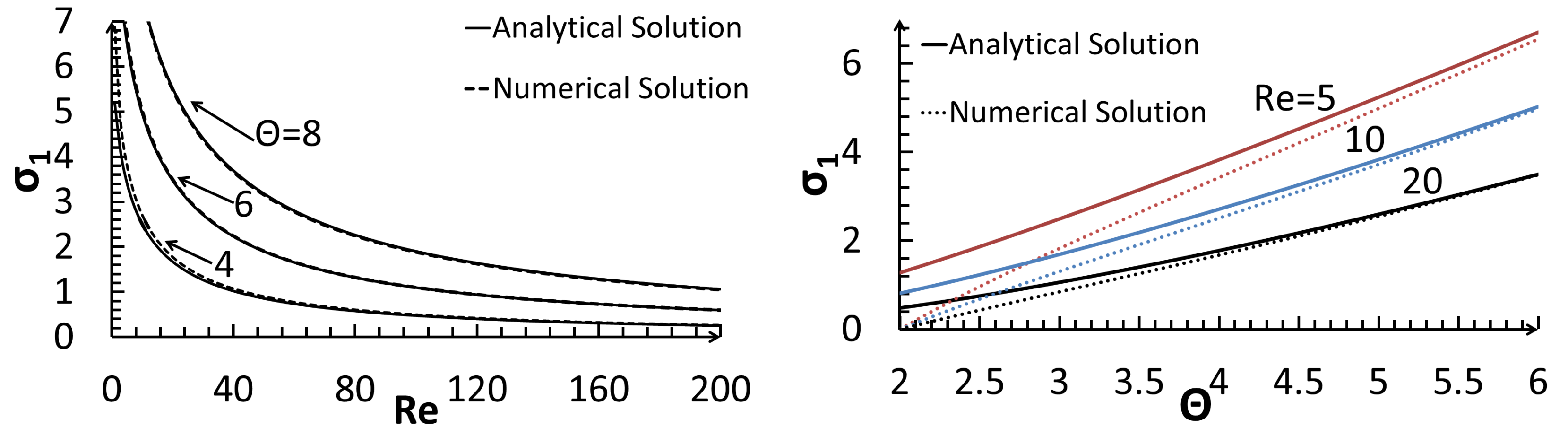

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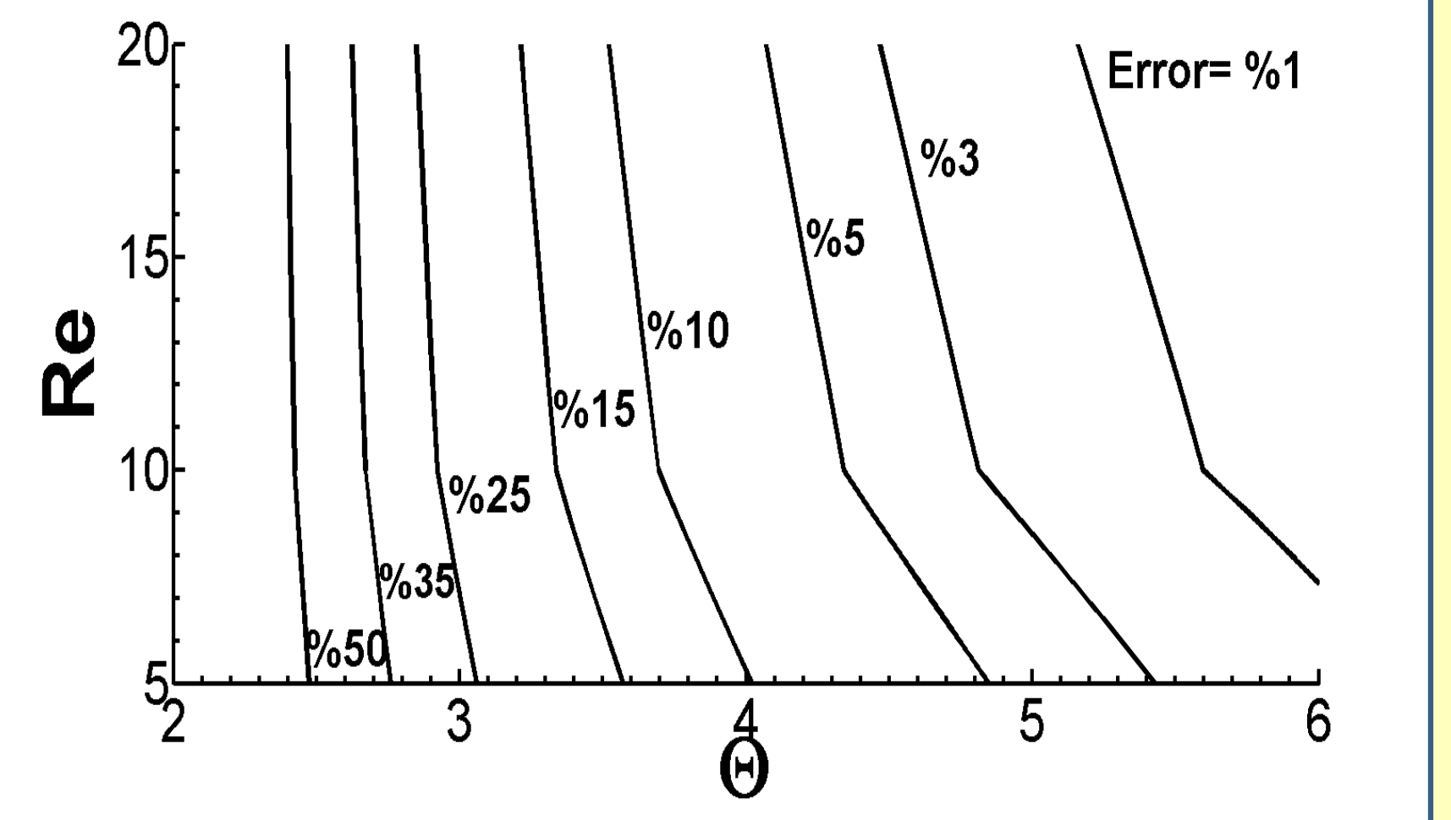
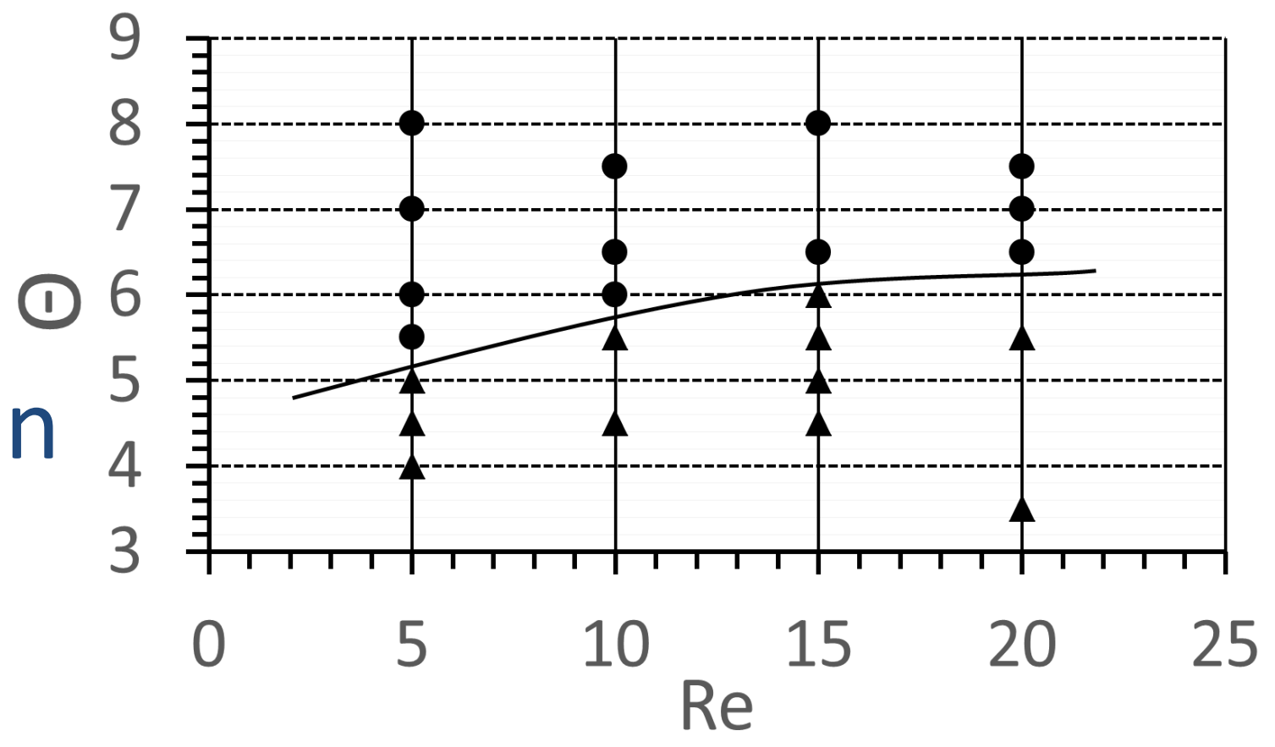


Contour scheme (the error isolines (in %)) demonstration in the Re-Θ diagram.

- Results above show that the 0th-order approximation is inaccurate, even for realistically large Re and/or Θ. At the same time, the 1st-order approximation is reasonably accurate for a wide range of parameters.

Conclusion

- Formulations [3,4] are revisited. Their intrinsic limitations are identified in the form of domains in a Re-Θ diagram. While the formulations are accurate for large Re and Θ, the accuracy deteriorates at other conditions. Finally, this analysis is supported by numerical simulations; see the figure on the right. Here, the exponential (circles) regime of flame acceleration is separated from a non-exponential regime (triangles) by the solid line associated with a threshold thermal expansion ratio.



Acknowledgements

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