Analysis of Ethylene-Oxygen Combustion in Micro-Pipes

Berk Demirgok[1], Orlando Ugarte[1], Damir Valiev[2], Chung K. Law[2], Ming-Hsun Wu[3], Vitaly Bychkov[4], **V'yacheslav Akkerman[1]***

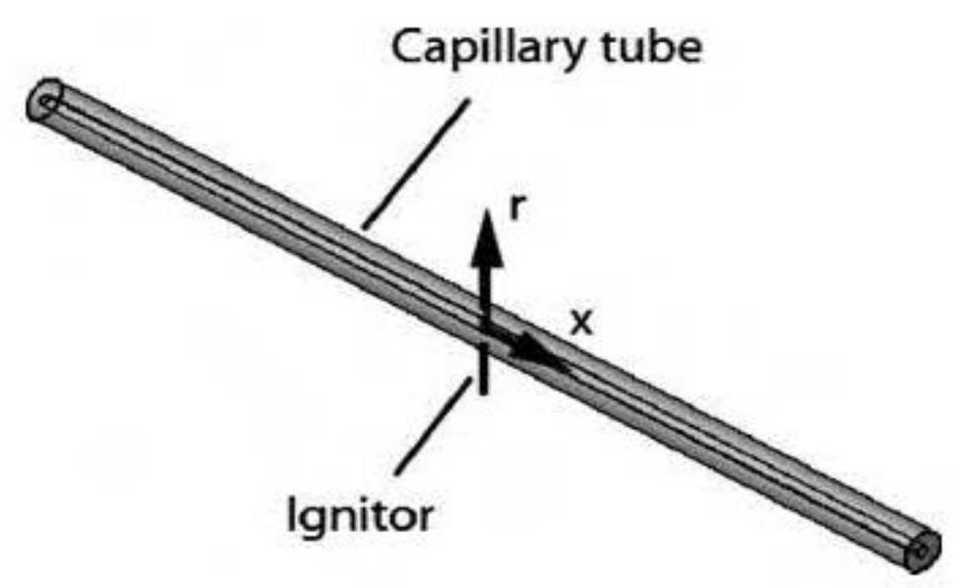
[1]Department of Mechanical and Aerospace Engineering, West Virginia University, [2]Department of Mechanical and Aerospace Engineering, Princeton University,
[3]Department of Mechanical Engineering, National Cheng Kung University, Taiwan, [4]Department of Physics, Umea University, Sweden
* **Corresponding author: Vyacheslav.Akkerman@mail.wvu.edu**

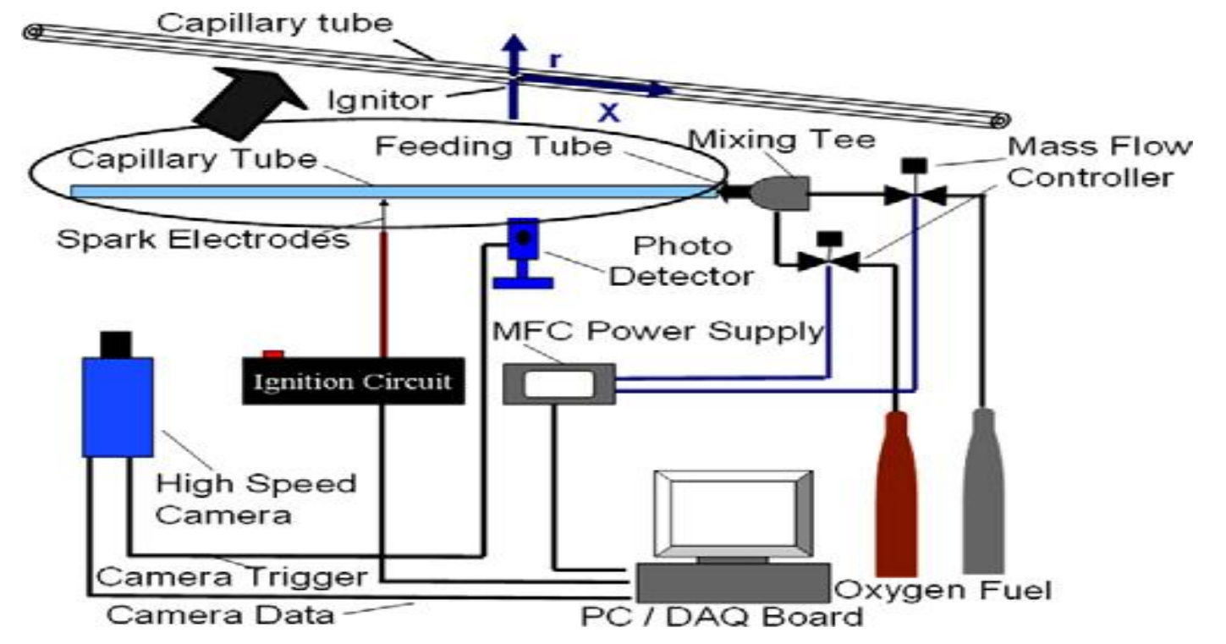
Motivation

- A flame (deflagration) front can accelerate spontaneously, possibly followed by a deflagration-to-detonation transition (DDT) event. The effect is extremely strong in tubes and channels, where the acceleration mechanism is associated, in particular, with wall friction. Practical applications include pulse detonation engines as well as the fire safety issues in mines, subways and power plants.
- In this work, the acceleration of premixed stoichiometric ethylene-oxygen (C_2H_4/O_2) flames in pipes of sub/near-millimeter radii is investigated computationally, analytically and experimentally.
- Specifically, the study aims to bridge the gap between the existing experimental measurements and theoretical formulation by means of numerical simulations.

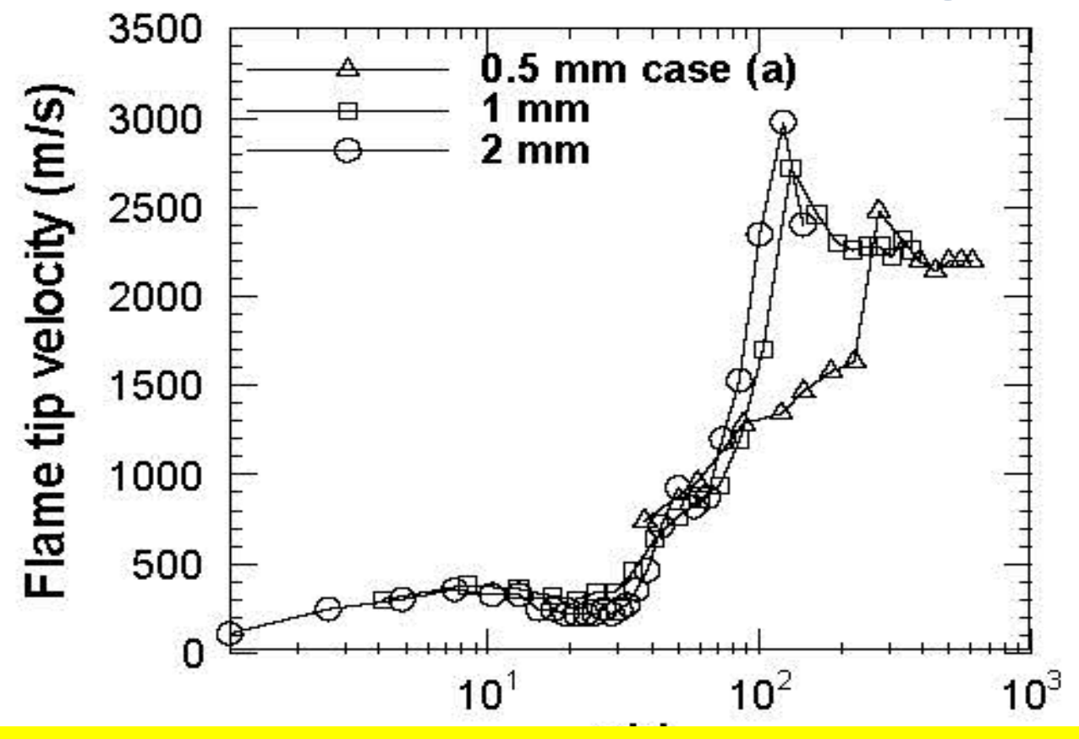
Experiments

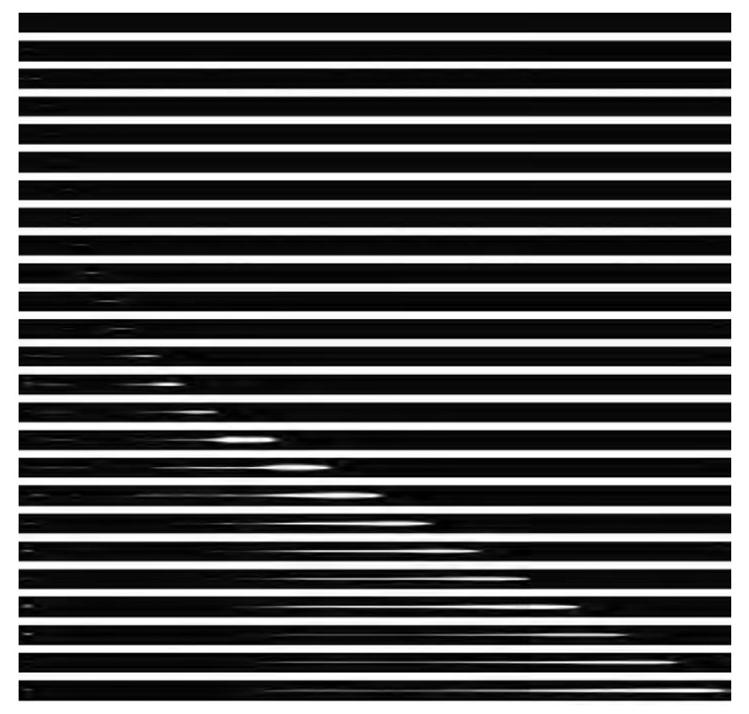
- C_2H_4/O_2 flame acceleration in capillary pipes with inner radii 0.25, 0.5, 1 and 1.5 mm was analyzed experimentally using high speed cinematography [1].
- The flame was ignited at the center of a 1.5 m long smooth tube under atmospheric pressure and room temperature.
- The effects of the Reynolds number and the equivalence ratio were investigated.





Schematic of experimental setup

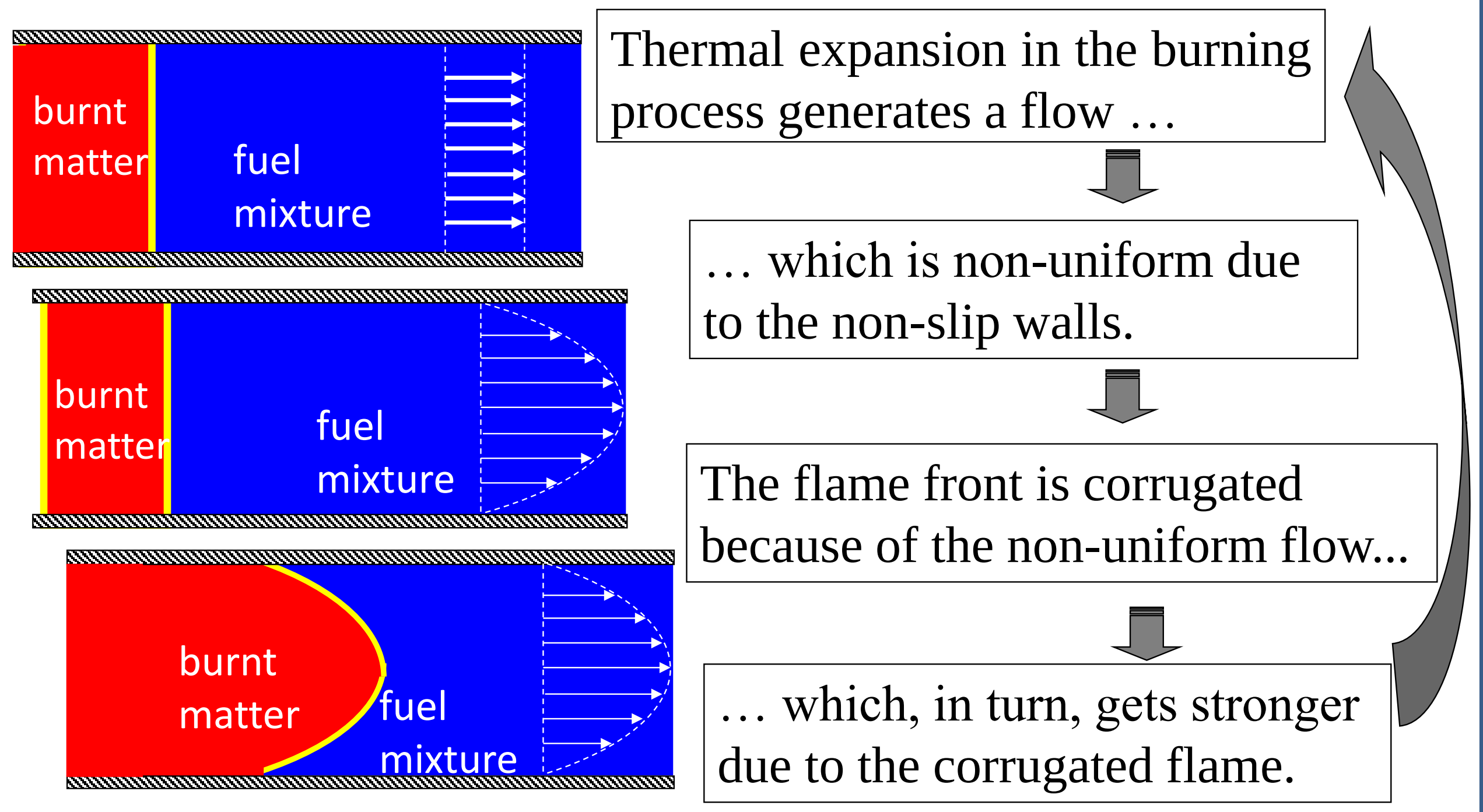




Evolution of flame shape

Mechanism of Flame Acceleration due to Wall Friction

- A combustible gas expands in the process of burning, thereby generating a flow. Due to non-slip conditions at the wall, the induced flow is highly non-uniform, which in turn causes the flame shape to be corrugated. Hence, the total burning rate grows, thereby inducing strong acceleration of the flame front, and the positive feedback between the flame and the flow [2-5].



Thermal expansion in the burning process generates a flow …

… which is non-uniform due to the non-slip walls.

The flame front is corrugated because of the non-uniform flow...

… which, in turn, gets stronger due to the corrugated flame.

Analytical Theories on Flame Acceleration in Tubes

a) Incompressible analytical theory [5]

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\nabla P + \nu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right), \quad z(r,t) = Z_{tip}(t) - F(r,t)$$

$$\langle u_z \rangle = (\Theta - 1)S_T, \qquad \Theta = \rho_u / \rho_b$$

$$\frac{\partial F}{\partial t} = u_z(0,t) - u_z(r,t) + 1 - \sqrt{1 + (\nabla F)^2}$$

Basic equations of the formulation

At this stage, theoretical/computational studies show exponential acceleration:

$$S_T / S_L \propto \exp(\sigma S_L t / R), \quad \sigma = \sigma(\mathrm{Re}) \to \Theta^2 / \mathrm{Re}.$$

b) Analytical theory accounting for a slowdown of the acceleration due to gas compressibility

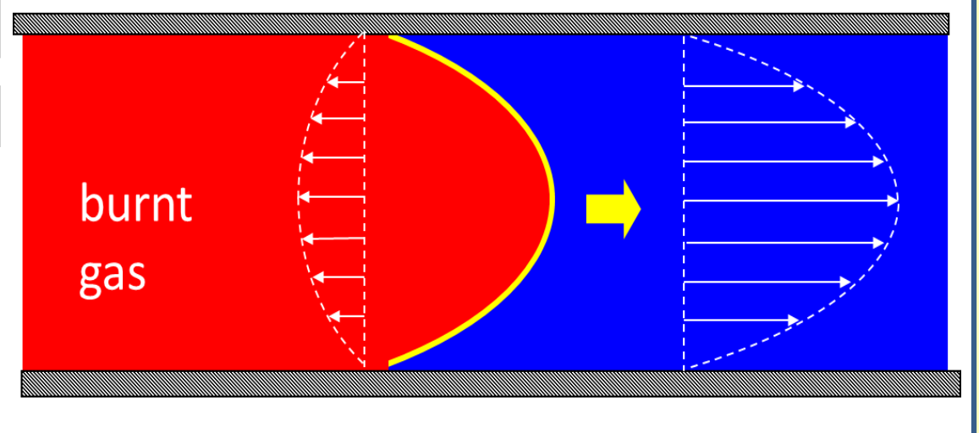
✓ Reduction in thermal expansion

✓ Backward flow in the burnt gas

$$\langle u_z \rangle = \frac{\Theta - 1}{\Theta} S_T \left[\dot{\Sigma} - (\gamma - 1) Ma \frac{\Theta - \tilde{m}}{\Theta^2} (\dot{\Sigma})^2 - \frac{Ma}{\Theta}\ddot{\Sigma}\Sigma\right]$$

Corrections

$$\tilde{m} \equiv m_u / m_b \qquad Ma \equiv S_L / c_0, \qquad \dot{\Sigma} = U_L / S_L,$$



Description of the Numerical Simulations

$$\frac{\partial}{\partial t}\rho + \frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r) + \frac{\partial}{\partial z}(\rho u_z) = 0$$

$$\frac{\partial}{\partial t}(\rho u_r) + (\mathbf{u}\cdot\nabla)(\rho u_r) - \rho\frac{u_\theta^2}{r} = -\frac{\partial P}{\partial r} + \zeta\left(\nabla^2 u_r - \frac{u_r}{r^2}\right)$$

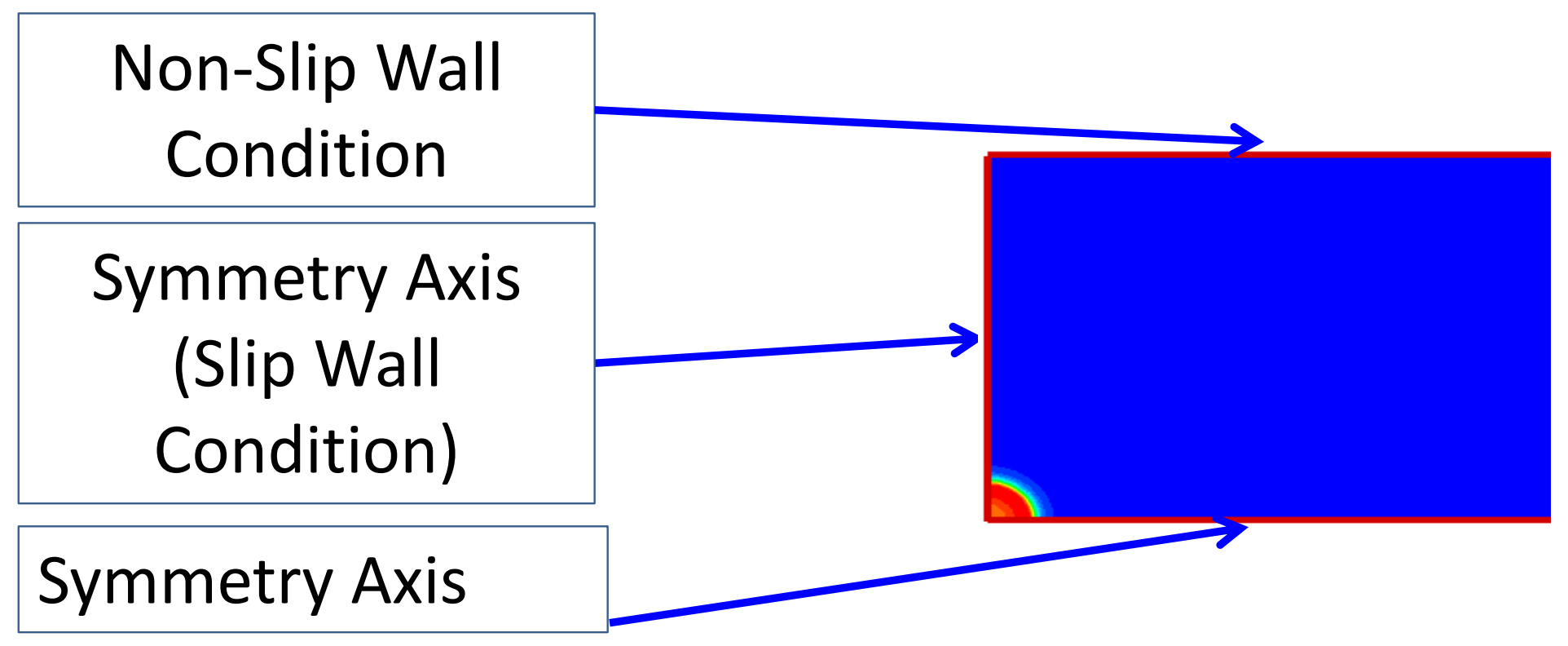
$$\frac{\partial}{\partial t}(\rho u_\theta) + (\mathbf{u}\cdot\nabla)(\rho u_\theta) + \rho\frac{u_\theta u_r}{r} = -\frac{\partial P}{\partial r} + \zeta\left(\nabla^2 u_\theta - \frac{u_\theta}{r^2}\right)$$

$$\frac{\partial}{\partial t}(\rho u_z) + (\mathbf{u}\cdot\nabla)(\rho u_z) = -\frac{\partial P}{\partial z} + \zeta\nabla^2 u_z$$

$$\frac{\partial}{\partial t}\left(\rho\varepsilon + \frac{1}{2}\rho\mathbf{u}^2\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(\rho r u_r h + \frac{r}{2}\rho u_r\mathbf{u}^2 + rq_r - ru_r\gamma_{r,r} - ru_z\gamma_{r,z} - ru_\theta\gamma_{r,\theta}\right)$$
$$+\frac{\partial}{\partial z}\left(\rho u_z h + \frac{1}{2}\rho u_z\mathbf{u}^2 + q_z - u_z\gamma_{z,z} - u_r\gamma_{r,z} - u_\theta\gamma_{z,\theta}\right) = 0$$

$$\frac{\partial}{\partial t}(\rho Y) + \frac{1}{r}\frac{\partial}{\partial r}\left(r\rho u_r Y - r\frac{\zeta}{\mathrm{Sc}}\frac{\partial Y}{\partial r}\right) + \frac{\partial}{\partial z}\left(\rho u_z Y - \frac{\zeta}{Sc}\frac{\partial Y}{\partial z}\right) = -\frac{\rho Y}{\tau_R}\exp(-E_a/R_p T)$$

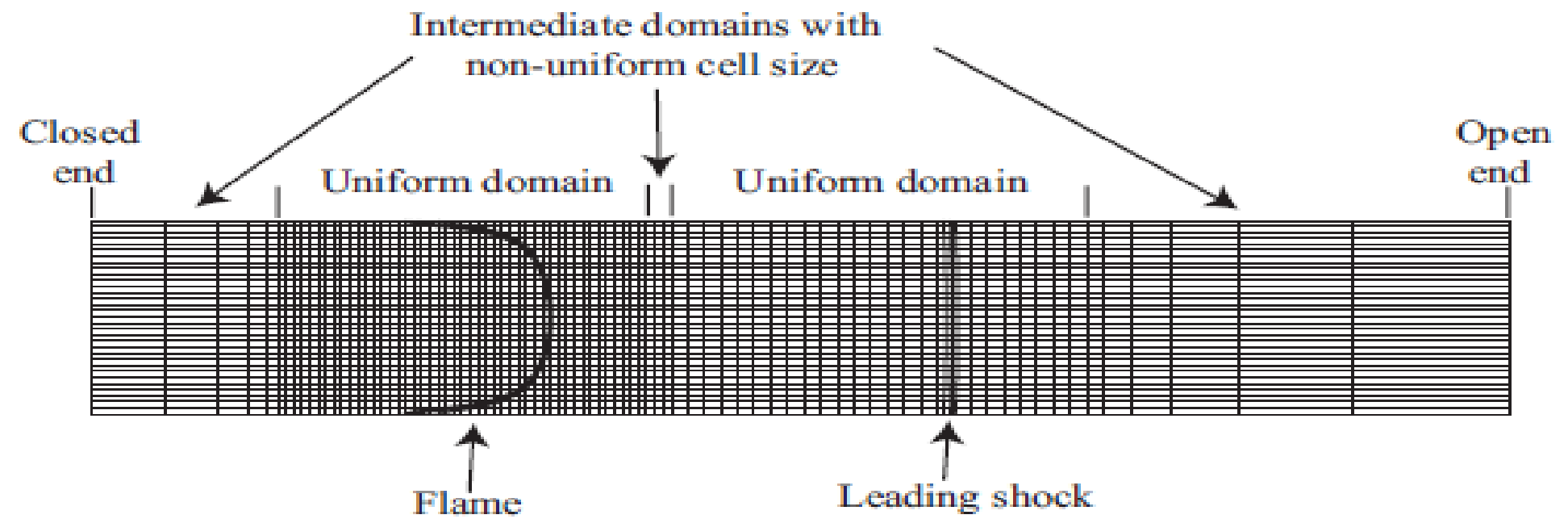
Basic equations of the simulations



Non-Slip Wall Condition

Symmetry Axis (Slip Wall Condition)

Symmetry Axis

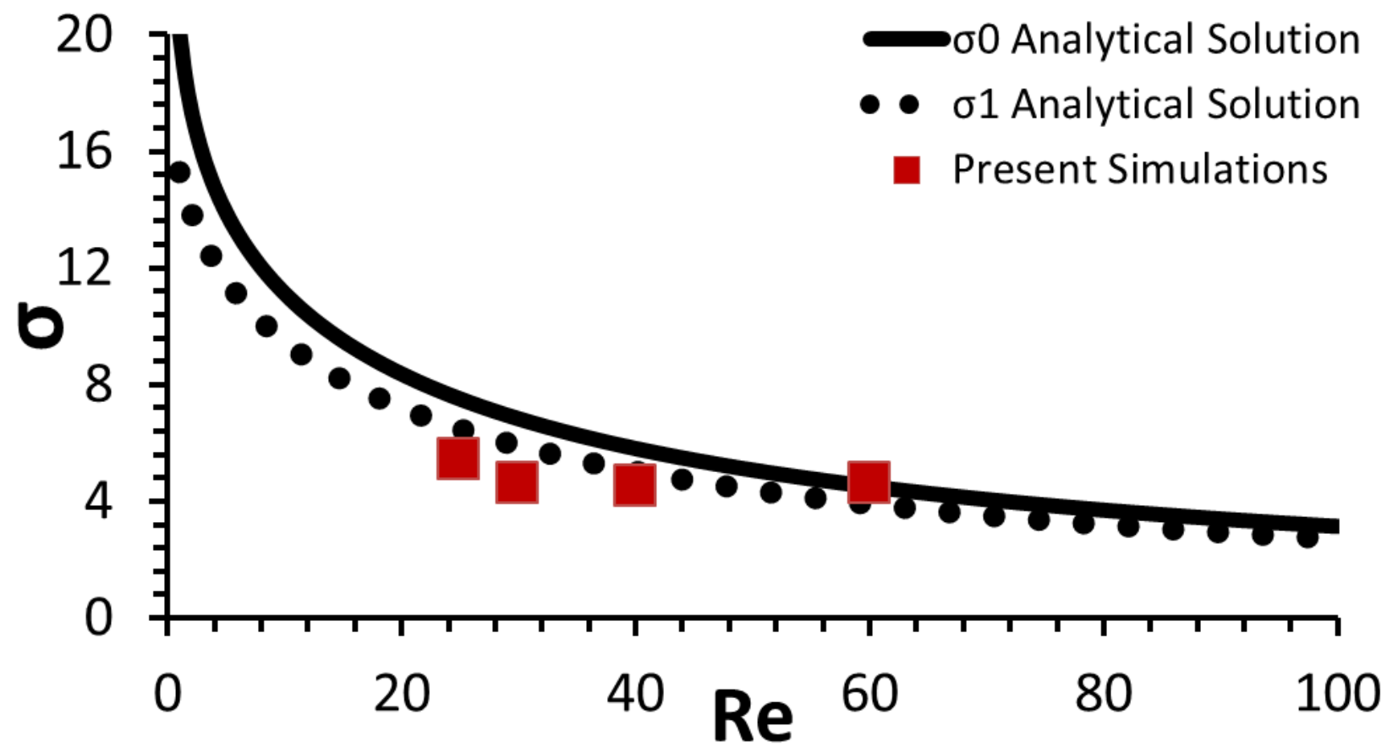


Adaptive grid used in our simulations

✓ The code is based on a cell-centered finite-volume scheme and adapted for block-structured grid system.

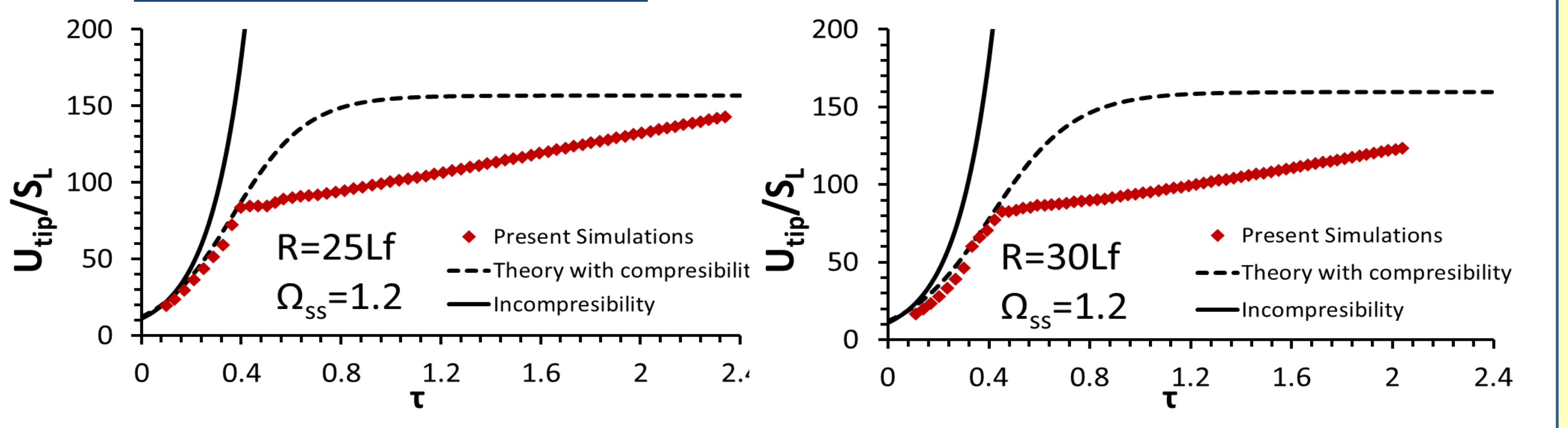
✓ The scheme is of the 2nd -order accuracy in time and the 4th -order in space for convective terms, and the 2nd -order in space for diffusive terms. The code is robust and adapted for parallel computations.

Results



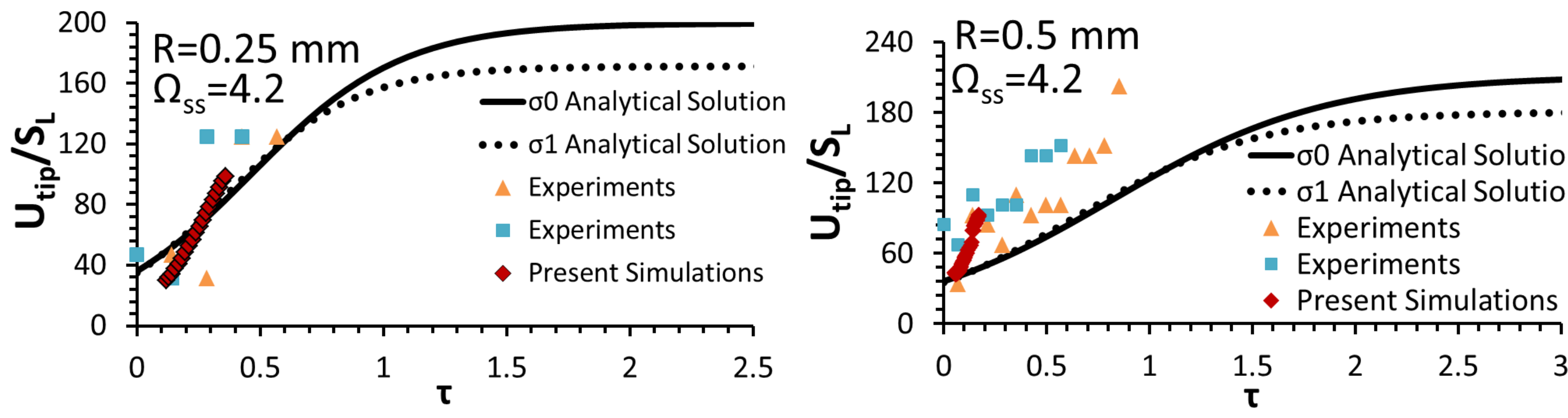
The flame acceleration rate σ versus the flame propagation Reynolds number Re at a fixed thermal expansion ratio Θ=10.6.

Theory vs Simulations

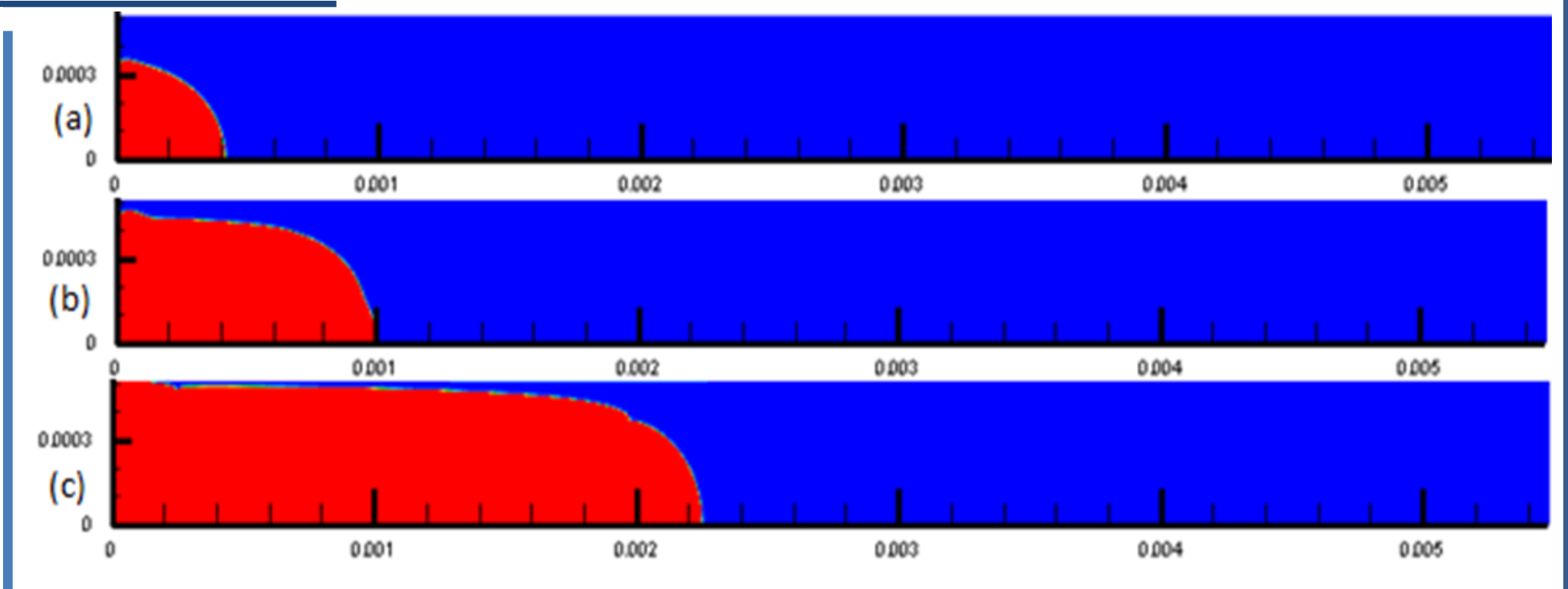


The scaled flame tip velocity U_{tip}/S_L versus scaled time τ at fixed $R=25L_f$ (left); and $R=30L_f$ (right).

Theory vs Simulations vs Experiments



The scaled flame tip velocity U_{tip}/S_L versus scaled time τ at fixed R=0.25 mm (left); and R=0.5 mm (right)



Evolution of the flame shape in a tube of radius R =0.25mm. The colors represent the temperature from 300K in the fuel mixture (blue) to the corresponding burnt matter temperature (red)

Summary

- The main flame characteristics (shape, velocity, flame tip locus, acceleration rate, flame-generated flow) are identified.
- Agreement between the simulations, theory and experiments is observed at the initial stage of the flame acceleration. The simulations bridge the gap between the experiments and the theory.

References

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Envelope